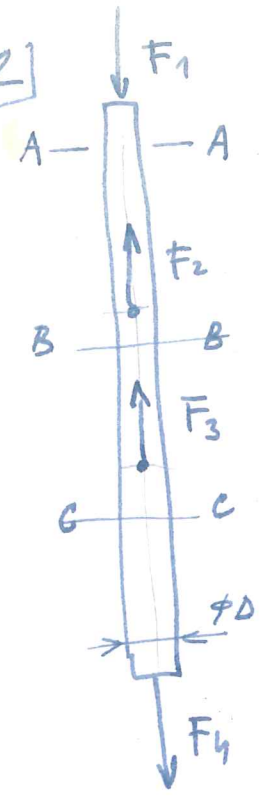
4.2



$F_1 = 1000\,N$
$F_2 = 3000\,N$
$F_3 = 2000\,N$
$F_4 = ?$

$\Sigma F_x = 0$ .... *

$\Sigma F_y = 0 \quad -F_1 + F_2 + F_3 - F_4 = 0$

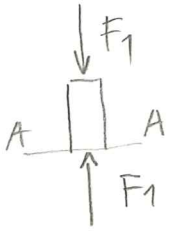
$-1000 + 3000 + 2000 - F_4 = 0$

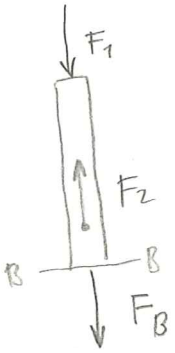
$F_4 = 4000\,N$

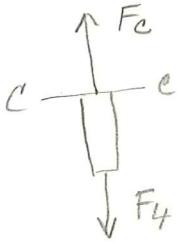
$F_A = |F_1| = 1000\,N$ ~~tah~~ tlak

$F_B = F_2 - F_1 = +2000\,N$ tah

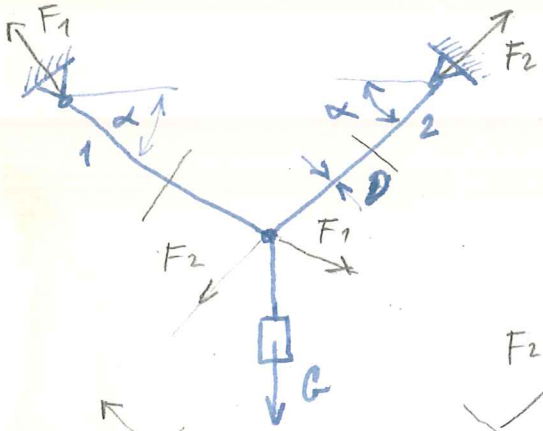
$F_C = F_4 = 4000\,N$ tah

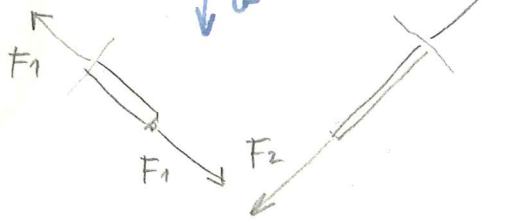




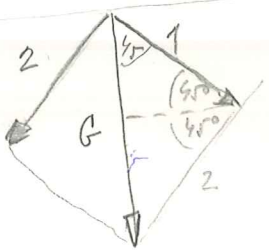


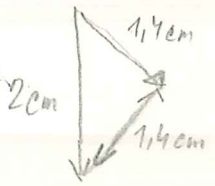
4.3





$\alpha = 45°$





$\frac{F_1}{G} = \cos\alpha$

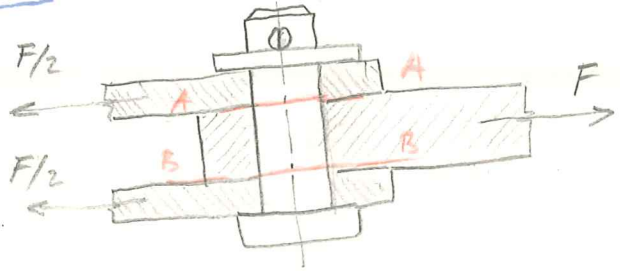
$F_1 = F_2 = G \cdot \cos\alpha =$

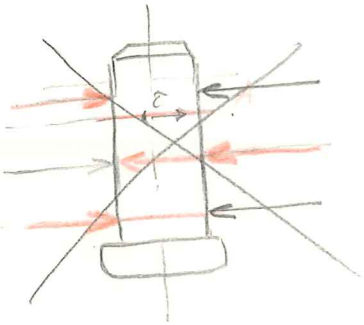
$= 1000 \cdot \cos 45° = \underline{707\,N}$

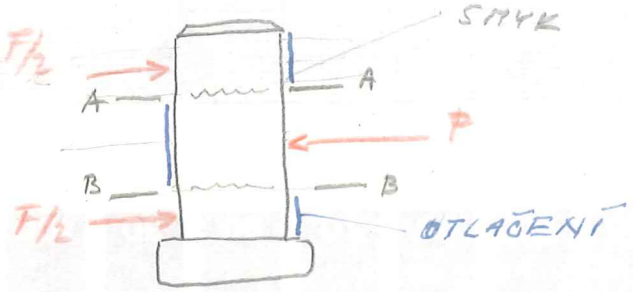
Namáhání prutu 1 a 2 je tahem (prostým)

4.4

$F = 8000\,N$



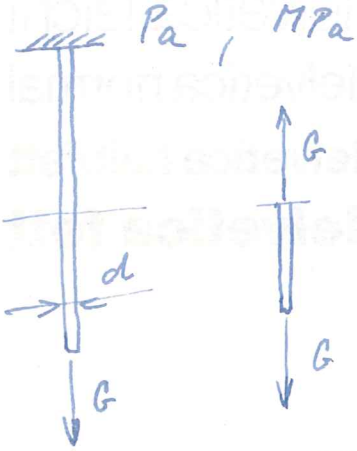




$\frac{F}{2} = 4000\ N$

4.5) $d = 2\ mm$, $G = 80\ N$

Pa , $MPa = ?$



$$\sigma = \frac{Q}{S} = \frac{80}{\frac{\pi d^2}{4}} = \frac{80}{\pi} = 25,46\ [N.mm^{-2}]$$

$$= 25,46\ MPa$$

$$= 2,546 \cdot 10^7\ Pa$$

4.6) $D = 6\ mm$

$A-A$ ... $\sigma_{tA} = \frac{F_A}{\frac{\pi d^2}{4}} = \frac{1000}{28,27} = 35,37\ MPa$

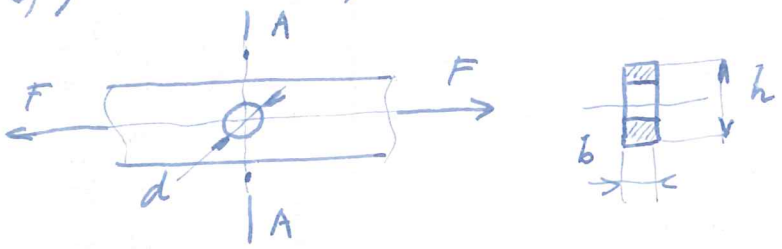
$B-B$ ... $\sigma_{tB} = \frac{F_B}{S} = \frac{2000}{28,27} = 70,74\ MPa$

$C-C$ ... $\sigma_{tC} = \frac{F_C}{S} = \frac{4000}{28,27} = 141,48\ MPa$

4.7) $b = 15\ mm$, $h = 30\ mm$, $F = 45\ kN$

a) jaké napětí bude v průřezu

b) jaké bude napětí v místě otvoru $d = 10\ mm$



a) $\sigma = \frac{F}{S} = \frac{45 \cdot 10^3}{15 \cdot 30} = 100\ MPa$

b) $\sigma_{max} = \frac{F}{S_1} = \frac{F}{b(h-d)} = \frac{45 \cdot 10^3}{15(30-10)} = 150\ MPa$

4.8 Tahlo složené ze dvou profilů ⌊ 80 × 80 × 10 ČSN 425510 je zatíženo tahovou silou $F = 270\,000\ N$

$\sigma = ?$

Tab. $S = 15{,}1\ cm^2 = 15{,}1 \cdot 10^2\ mm^2$

$$\sigma = \frac{F}{2S} = \frac{270\,000}{2 \cdot 15{,}1 \cdot 10^2} = 89{,}4\ MPa$$

4.9 Sestava obr. 4.6 str. 77 $l_1 = 200\ mm$ $b = 10\ mm$ $h_2 = 16\ mm$ $l_2 = 400\ mm$ $h_1 = 50\ mm$

$D = 48\ mm$, $d = 32\ mm$, $D_1 = D_2 = 5\ mm$, $D_3 = 4\ mm$, $D_4 = 10\ mm$

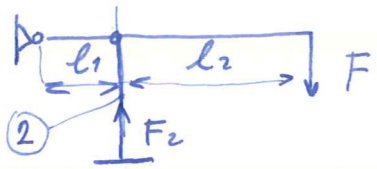
$F = 1000\ N$

a) napětí v dříku sloupku

b) napětí v průřezu čepu 4 a tlak ve styk. ploše

c) tlak ve styk. ploše sedla čepu

d) napětí v průřezu tahla



$$F(l_1 + l_2) = F_2 \cdot l_1$$

$$F_2 = F\,\frac{l_1 + l_2}{l_1} = 1000\,\frac{200 + 400}{200} = 3000\ N$$

a) sloupek ② napětí $\quad \sigma_d = \frac{F_2}{S} = \frac{F_2}{h_2 \cdot b} = \frac{3000}{16 \cdot 10} = 18{,}75\ MPa$

b) napětí v čepu ④ a tlak ve styk. pl.

$$\tau = \frac{F_2}{2S} = \frac{F_2}{2\,\frac{\pi D_4^2}{4}} = \frac{2F_2}{\pi \cdot 10^2} = \frac{2 \cdot 3000}{\pi \cdot 100} = 19{,}09\ MPa$$

$$p = \frac{F_2}{b \cdot D_4} = \frac{3000}{10 \cdot 10} = 30\ MPa$$

c) tlak ve styk. pl. sedla

$$p = \frac{F_2}{S_s} = \frac{F_2}{\frac{\pi}{4}(D^2 - d^2)} = \frac{3000}{\frac{\pi}{4}(48^2 - 32^2)} = 2{,}98\ MPa$$

d) napětí v tahle ③

$$\sigma = \frac{F}{S_t} = \frac{F}{\frac{\pi D_3^2}{4}} = \frac{4 \cdot 1000}{\pi \cdot 4^2} = 79{,}57\ MPa$$

4.10 Kruh. tahlo $d_0 = 20\,mm$ a délky $l_0 = 1\,m$.
Po zatížení $F = 130\,000\,N$ má délku $l = 1{,}002\,m$

Urči: a) velikost napětí
b) celk. prodloužení
c) poměrné prodl. tahla

a) $\sigma = \frac{F}{S} = \frac{F}{\frac{\pi d_0^2}{4}} = \frac{4 \cdot 130000}{\pi \cdot 20^2} = \underline{413{,}8\,MPa}$

b) $\Delta l = l - l_0 = 1002 - 1000 = \underline{2\,mm}$

c) $\varepsilon = \frac{\Delta l}{l_0} = \frac{2}{1000} = \underline{0{,}002}$

4.11 $\sigma_{kt} = ?$ $\sigma_{pt} = ?$ Tyč $S_0 = 28 \times 12\,mm$

z diagramu $F_k = 9{,}3 \cdot 10^4\,N$

$F_p = 17 \cdot 10^4\,N$

$\sigma_{kt} = \frac{F_k}{S_0} = \frac{9{,}3 \cdot 10^4}{28 \cdot 12} = \underline{276\,MPa}$

$\sigma_{pt} = \frac{F_p}{S_0} = \frac{17 \cdot 10^4}{28 \cdot 12} = \underline{506\,MPa}$

4.12 NE

4.13 Tyč 1 ... S_0, l_1 $F_1 = F_2$ $l_{01} = 600\,mm$; $l_1 = 609\,mm$
Tyč 2 ... S_0, l_2 $l_{02} = 1000\,mm$; $l_2 = 1010\,mm$

Urči rozdíl v %

$\varepsilon_1 = \frac{l_1 - l_{01}}{l_{01}} = \frac{609 - 600}{600} = \underline{0{,}015}$ $\varepsilon_1 = 1{,}5\,\%$

$\varepsilon_2 = \frac{l_2 - l_{02}}{l_{02}} = \frac{1010 - 1000}{1000} = \underline{0{,}01}$ $\varepsilon_2 = 1{,}0\,\%$

Rozdíl je $\underline{0{,}5\,\%}$

4.14 $d_0 = 20\,mm$, $l_0 = 200\,mm$ $\quad \delta = ?$

$d = 11{,}8\,mm$, $l = 260\,mm$ $\quad \psi = ?$

$$\delta = \frac{l - l_0}{l_0} \cdot 100 = \frac{260 - 200}{200} \cdot 100 = \underline{30\,\%}$$

$$\psi = \frac{S_0 - S}{S_0} = \frac{\frac{\pi}{4}(S_0^2 - S^2)}{\frac{\pi}{4} S_0^2} \cdot 100 = \frac{20^2 - 11{,}8^2}{20^2} \cdot 100 = \underline{65{,}2\,\%}$$

4.15 $\delta = ?$ z diagramu obr. 4.7 str. 78

$$\Delta l = \frac{1}{35}\,mm \; ; \; l_0 = 120\,mm$$

$$\delta = \frac{\Delta l}{l_0} \cdot 100 = \frac{70 \cdot \frac{1}{35}}{120} \cdot 100 = \underline{1{,}66\,\%}$$

4.16 Krátký pásek $d = 20\,mm$; $\sigma_{Pt} = 1300\,MPa$ $\quad F = ?$ tah se pásek roztrhl.

$$F = \sigma_{Pt} \cdot S = \sigma_{Pt} \frac{\pi d^2}{4} = 1300 \frac{\pi \cdot 20^2}{4} = \underline{408\,393\,N}$$

4.17 Ocelový kruhový průřez má $l = 800\,mm$

$\Delta l = ?$ tah napětí v tyči $\sigma_t = 60\,MPa$

$\varepsilon = ?$ a $E = 2{,}1 \cdot 10^5\,MPa$

$$\sigma = E \cdot \varepsilon \rightarrow \varepsilon = \frac{\sigma}{E} = \frac{60}{2{,}1 \cdot 10^5} = 28{,}57 \cdot 10^{-5} = \underline{0{,}00285}$$

$$\varepsilon = \frac{\Delta l}{l_0} \rightarrow \Delta l = \varepsilon \cdot l_0 = 0{,}00285 \cdot 800 = \underline{2{,}28\,mm}$$

4.18 Určete modul pružnosti v tahu z diagr. obr. 4.7 str. 78

$F_u = 8{,}5 \cdot 10^4\,N$ $\quad S_0 = 336\,mm^2$

$$\Delta l = \frac{5}{35} = 0{,}1428\,5\,mm$$

$$\varepsilon = \frac{\Delta l}{l_0} = \frac{0{,}1428\,5}{120} = 1{,}1904 \cdot 10^{-3}$$

$$\sigma_u = \frac{F_u}{S_0} = \frac{8{,}5 \cdot 10^4}{336} = 252{,}9\,MPa \qquad \sigma = E \cdot \varepsilon$$

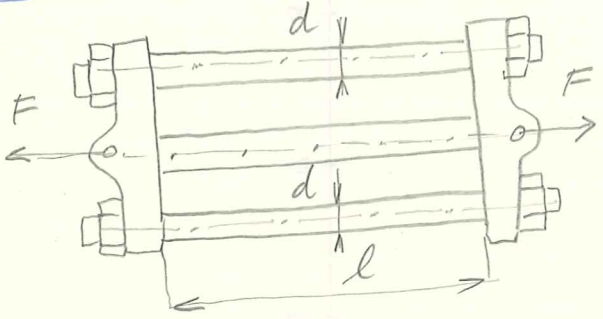
$$E = \frac{\sigma_u}{\varepsilon} = \frac{252{,}9}{1{,}1904 \cdot 10^{-3}} = 212\,449{,}5\,MPa = \underline{2{,}124 \cdot 10^5\,MPa}$$

4.19 Tahlo obdélníkové $b_0 = 5\,mm$, $h_0 = 60\,mm$, $l_0 = 1{,}2\,m$
Při zatížení silou F je $\Delta l = 0{,}6\,mm$; $E = 2{,}0 \cdot 10^5\,MPa$
$F = ?$

$$\sigma = E \cdot \varepsilon = E \frac{\Delta l}{l_0} = 2{,}0 \cdot 10^5 \frac{0{,}6}{1200} = 100\,MPa$$

$$F = \sigma \cdot S_0 = 100 \cdot b_0 h_0 = 100 \cdot 5 \cdot 60 = 30\,000\,N$$

4.20



$F = ?$ (oddálení odlitků, aby bylo možné vložit tyč)
$l = 200{,}2\,mm$
Původní vzdálenost $l_0 = 200\,mm$
$d_0 = 10\,mm \dots E = 2{,}0 \cdot 10^5\,MPa$

$$\Delta l = l - l_0 = 200{,}2 - 200 = 0{,}2\,mm$$

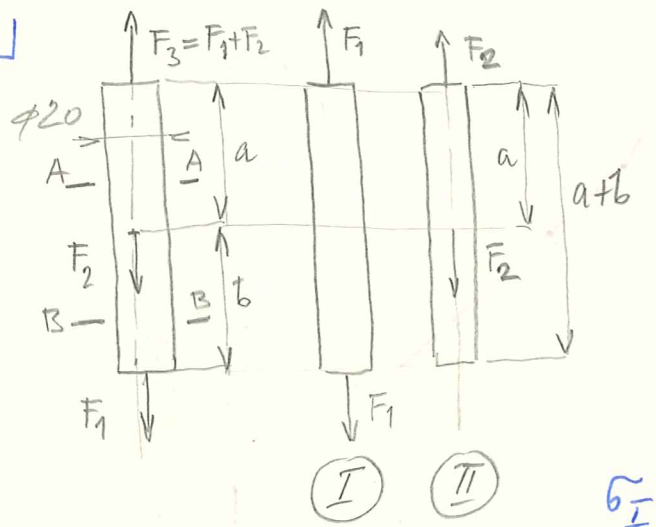
$$\varepsilon = \frac{\Delta l}{l_0} = \frac{0{,}2}{200} = 0{,}001$$

$$\sigma = \frac{F}{2S_0} = E \cdot \varepsilon \Longrightarrow F = E\varepsilon 2S_0 = E \cdot \varepsilon \cdot 2\frac{\pi d_0^2}{4} =$$

$$= E\varepsilon \frac{\pi}{2} d_0^2 = 2{,}0 \cdot 10^5 \cdot 0{,}001 \cdot \frac{\pi}{2} 10^2 =$$

$$= 31400\,N = 31{,}4\,kN$$

4.21



$F_1 = 32\,kN$, $F_2 = 28\,kN$; $F_3 = ?$
$E = 2{,}1 \cdot 10^5\,MPa$ superpozice
$a = 600\,mm$
$b = 800\,mm$
$d = 20\,mm$

$$F_3 = F_1 + F_2 = 32 + 28 = 60\,kN$$

$$\sigma_I = \frac{F_1}{S_0} = \frac{F_1}{\frac{\pi d^2}{4}} = \frac{4F_1}{\pi d^2} = \frac{4 \cdot 32000}{\pi \cdot 20^2} = 101{,}85$$

$$\sigma_{II} = \frac{F_2}{S_0} = \frac{4F_2}{\pi d^2} = \frac{4 \cdot 28000}{\pi \cdot 20^2} = 89{,}12\,MPa$$

$$A-A \dots \sigma_A = \sigma_I + \sigma_{II} = 101{,}85 + 89{,}12 = 190{,}97\,MPa$$

$$B-B \dots \sigma_B = \sigma_{II} = 89{,}12\,MPa$$

4.21 | Pokračování

I ... $\sigma_I = E\varepsilon_I \Rightarrow \varepsilon_I = \frac{\sigma_I}{E} = \frac{101,85}{2,1.10^5} = 4,85.10^{-4}$

$\varepsilon_I = \frac{\Delta l_I}{l_0} \Rightarrow \Delta l_I = \varepsilon l_0 = 4,85.10^{-4}(a+b) = 4,85.10^{-4}(600+800)$

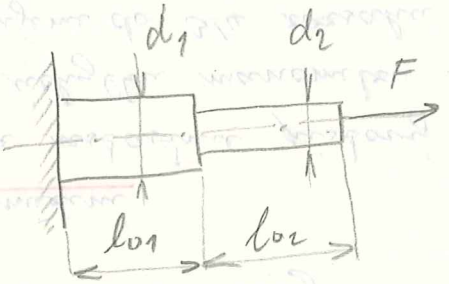
$= 0,679\ mm$

II ... $\sigma_{II} = E.\varepsilon_{II} \Rightarrow \varepsilon_{II} = \frac{\sigma_{II}}{E} = \frac{89,12}{2,1.10^5} = 4,243.10^{-4}$

$\Delta l_{II} = \varepsilon_{II} l_a = 4,243.10^{-4}.600 = 0,254\ mm$

$\Delta l = \Delta l_I + \Delta l_{II} = 0,679 + 0,254 = 0,933\ mm$

4.22 |



$F = 40\,000 N$ $\quad d_1 = 40 mm,\ l_{01} = 400 mm$

$E = 2,0.10^5 MPa$ $\quad d_2 = 20 mm,\ l_{02} = 800 mm$

$\Delta l_{celk} = ?$ superpozice

$\Delta l_{celk} = \Delta l_1 + \Delta l_2$

$\sigma_1 = \frac{F}{S_1} = \frac{F}{\frac{\pi}{4} d_1^2} = \frac{4F}{\pi d_1^2} = \frac{4.40000}{\pi.40^2} = 31,8 MPa$

$\sigma = E\varepsilon \longrightarrow \varepsilon_1 = \frac{\sigma_1}{E} = \frac{31,8}{2.10^5} = 1,59.10^{-4}$

$\Delta l_1 = \varepsilon_1 l_{01} = 1,59.10^{-4}.400 = 0,0636\ mm$

$\sigma_2 = \frac{F}{S_2} = \frac{4F}{\pi d_2^2} = \frac{4.40\,000}{\pi.20^2} = 127,3 MPa$

$\varepsilon_2 = \frac{\sigma_2}{E} = \frac{127,3}{2.10^5} = 6,365.10^{-4}$

$\Delta l_2 = \varepsilon_2 l_{02} = 6,365.10^{-4}.800 = 0,5092\ mm$

$\Delta l_{celk} = \Delta l_1 + \Delta l_2 = 0,0636 + 0,5092 = 0,5728\ mm$

4.23 Tyč ocelová a měděná ϕ se stejnou délkou l_0 působí tah síla F stejná a tyče se prodloužily o stejné Δl

$d_2 = ?$ měděná tyč
$d_1 = 2\,mm$ ocel tyč

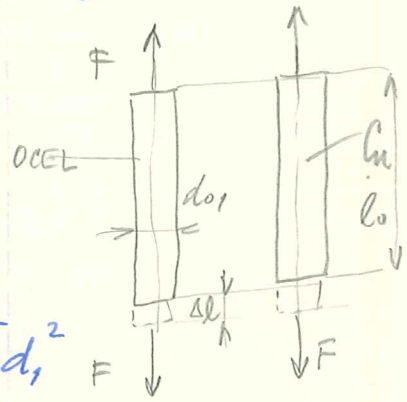
$E_{Cu} = 1{,}18 \cdot 10^5\ MPa$
$E_{ocel} = 2 \cdot 10^5\ MPa$

$\sigma_1 = E_o \cdot \varepsilon$ $\quad \sigma_2 = E_{Cu} \cdot \varepsilon$ $\quad \varepsilon_1 = \varepsilon_2$ ze zadání

$F_1 = F_2 = F$

$\frac{F_1}{S_1} = E_o \varepsilon$ $\quad \frac{F}{S_2} = E_{Cu} \cdot \varepsilon$

$\varepsilon = \frac{F}{E_o S_1}$ $\quad \varepsilon = \frac{F}{E_{Cu} S_2}$



$\frac{F}{E_o S_1} = \frac{F}{E_{Cu} S_2}$ .... $E_{Cu} S_2 = E_o S_1$

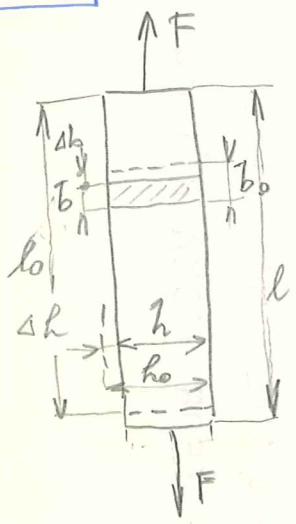
$E_{Cu} \frac{\pi}{4} d_2^2 = E_o \frac{\pi}{4} d_1^2$

$d_2^2 = \frac{E_o d_1^2}{E_{Cu}}$

$d_2 = d_1 \sqrt{\frac{E_o}{E_{Cu}}}$

$d_2 = 2\sqrt{\frac{2 \cdot 10^5}{1{,}18 \cdot 10^5}} = 2\sqrt{\frac{2}{1{,}18}} = 2{,}6\,mm$

4.24



$E = 2{,}0 \cdot 10^5\ MPa$
$l_0 = 100\,mm$
$\Delta l = 0{,}05\,mm$
$h_0 = 60\,mm$
$\Delta h = 0{,}0093\,mm$
$b_0 = 3\,mm$

$\mu = ?$
$m = ?$
$\sigma = ?$
$F = ?$

POZOR – ZMATEČNÉ ZADÁNÍ VE SBÍRCE

$\varepsilon_l = \frac{\Delta l}{l_0} = \frac{0{,}05}{100} = 5 \cdot 10^{-4}$

$\varepsilon_h = \frac{\Delta h}{h_0} = \frac{0{,}093}{60} = 1{,}55 \cdot 10^{-4}$

$\mu = \frac{\varepsilon_h}{\varepsilon_l} = \frac{5 \cdot 10^{-4}}{1{,}55 \cdot 10^{-4}} = 0{,}31$

$m = \frac{1}{\mu} = \frac{1}{0{,}31} = 3{,}225$

$\sigma = E \varepsilon_l = 2 \cdot 10^5 \cdot 5 \cdot 10^{-4} = 100\,MPa$

$F = \sigma \cdot S_0 = \sigma\, b_0 h_0 = 100 \cdot 3 \cdot 60 = 18000\,N$

4.25 Ocel tyč $l_0 = 4\,m$ ⌀ $b_0 = 40\,mm$; $h_0 = 50\,mm$
$F = 200\,000\,N$ prodloužila o $\Delta l = 2\,mm$. Urči:
a) napětí
b) změnu příčného délkového rozměru
c) původní změnu délky příčného rozměru, když $\mu = 0{,}3$

a) $\sigma = \frac{F}{b_0 h_0} = \frac{200\,000}{40 \cdot 50} = \underline{100\,MPa}$

b) $\varepsilon_z = \frac{\Delta l}{l_0} = \frac{2}{4000} = \underline{5 \cdot 10^{-4}}$

$\mu = \frac{\varepsilon_x}{\varepsilon_z} \Rightarrow \varepsilon_x = \varepsilon_y = \mu\,\varepsilon_z = 0{,}3 \cdot 5 \cdot 10^{-4} = \underline{1{,}5 \cdot 10^{-4}}$

c) $\varepsilon_x = \frac{\Delta h}{h_0} \Rightarrow \Delta h = \varepsilon_x \cdot h_0 = 1{,}5 \cdot 10^{-4} \cdot 50 = \underline{7{,}5 \cdot 10^{-3}\,mm}$

$\Delta b = \varepsilon_x b_0 = 1{,}5 \cdot 10^{-4} \cdot 40 = \underline{6 \cdot 10^{-3}\,mm}$

4.26 Ocelová tyč ⌀ $d_0 = 25\,mm$; $l_0 = 200\,mm$ ($\mu = ?$, $m = ?$)
$d = 24{,}943\,mm$; $\Delta l = 1{,}5\,mm$

$\varepsilon_z = \frac{\Delta l}{l_0} = \frac{1{,}5}{200} = 7{,}5 \cdot 10^{-3}$

$\varepsilon_x = \frac{\Delta d}{d_0} = \frac{25{,}00 - 24{,}943}{25} = 2{,}28 \cdot 10^{-3}$

$\mu = \frac{\varepsilon_x}{\varepsilon_z} = \frac{2{,}28 \cdot 10^{-3}}{7{,}5 \cdot 10^{-3}} = 0{,}304$

$m = \frac{1}{\mu} = 3{,}289$

4.27 $\Delta V = ?$ … $E = 2 \cdot 10^5\,MPa$
$\mu = 0{,}3$
$\sigma_t = 140\,MPa$

$\Delta V = \varepsilon\,(2\mu - 1)$ $\qquad \sigma = E \cdot \varepsilon \rightarrow \underline{\varepsilon} = \frac{\sigma}{E} = \frac{140}{2 \cdot 10^5} = \underline{7 \cdot 10^{-4}}$

$\Delta V = 7 \cdot 10^{-4}\,(2 \cdot 0{,}3 - 1) = \underline{-2{,}8 \cdot 10^{-4}}$

$\Delta V\,[\%] = -2{,}8 \cdot 10^{-4} \cdot 100 = \underline{0{,}028\,\%}$

4.28 Tahový diagram obr. 4.7 str. 78

$A_{def} = ?$
$w = ?$

$F_u = 85\ kN$
$\Delta l = \frac{5}{35}\ mm$
$S_0 = 336\ mm^2$
$l_0 = 120\ mm$
} z diagramu.

$$A_{def} = \frac{1}{2} F_u \Delta l = \frac{1}{2}\, 85000\, \frac{5}{35} \cdot 10^{-3} = 6{,}1\ J$$

$$w = \frac{A}{V} = \frac{6{,}1}{0{,}12 \cdot 336 \cdot 10^{-6}} = 1{,}51 \cdot 10^5\ J/m^3 = \underline{151 \cdot 10^3\ J/m^3}$$

4.29 Tyč $\square$ $h = 40\ mm$; $b = 30\ mm$; $l_0 = 2500\ mm$, $\Delta l = 0{,}05\ mm$
$E = 2{,}0 \cdot 10^5\ MPa$ $F = ?$ $A_{def} = ?$

$$\varepsilon = \frac{\Delta l}{l_0} = \frac{0{,}05}{2500} = 2 \cdot 10^{-4}$$

$$\sigma = E \cdot \varepsilon = 2{,}0 \cdot 10^5 \cdot 2 \cdot 10^{-4} = 40\ MPa$$

$$F = \sigma \cdot S = 40 \cdot b l = 40 \cdot 30{,}40 = \underline{48000\ N}$$

$$A_{def} = \frac{1}{2} F \Delta l = \frac{1}{2}\, 48000 \cdot 0{,}05 \cdot 10^{-3} = \underline{1{,}26\ J}$$

4.30 $\sigma_u' = 200\ MPa$; $E = 7{,}2 \cdot 10^4\ MPa$
$w = ?\ [J \cdot m^{-3}]$

$$w = \frac{1}{2} \frac{\sigma^2}{E} = \frac{1}{2} \frac{(200 \cdot 10^6)^2\ [Pa]}{7{,}2 \cdot 10^4 \cdot 10^6\ [Pa]} = \underline{27{,}7 \cdot 10^4\ J m^{-3}}$$

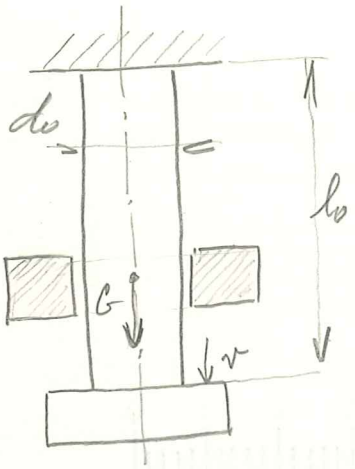
4.31 $\sigma_u' = 240\ MPa$; $E = 2 \cdot 10^5\ MPa$

$$w = \frac{1}{2} \frac{\sigma^2}{E} = \frac{1}{2} \frac{(240 \cdot 10^6)^2}{2 \cdot 10^5 \cdot 10^6} = \underline{14{,}4 \cdot 10^4\ J m^{-3}}$$

Houževnatost je u oceli menší, než u hliníkové slitiny

4.32 $d_0 = ?$ $\quad \sigma_t = 100\ MPa$, $E = 2{,}0 \cdot 10^5\ MPa$

$l_0 = 1\ m$ $\quad G = 200\ N$; $v = 4\ m\,s^{-1}$



$$E_k = \frac{1}{2} m v^2 = \frac{1}{2} \frac{G}{g} v^2 = \frac{1}{2} \frac{200}{9{,}81} \cdot 4^2 = \underline{163{,}09\ J}$$

$$E_k = A_{def}$$

$w_0 = 0$; $w_{max} = \frac{A_{def}}{V}$ ... $w_s = \frac{A_{def}}{2V}$ (stř. hodnota)

$$w = \frac{1}{2} \frac{\sigma^2}{E} = \frac{1}{2} \frac{(100 \cdot 10^6)^2}{2 \cdot 10^5 \cdot 10^6} = \underline{2500\ J \cdot m^{-3}}$$

$$w = \frac{A_{def}}{2V} \Rightarrow \underline{V} = \frac{A_{def}}{2w} = \frac{163{,}09}{2 \cdot 2500} = \underline{3{,}2618 \cdot 10^{-3}\ m^3}$$

$V = l \frac{\pi d_0^2}{4}$

$$d_0 = \sqrt{\frac{4V}{\pi l}} = \sqrt{\frac{4 \cdot 3{,}2618 \cdot 10^{-3}}{\pi \cdot 1}} = 0{,}0644\ m$$

$$\underline{\underline{d_0 = 64{,}4\ mm}}$$

4.33 ocel 11 340 $\quad \sigma_{Dt} = ?$ — statické, míjivé, střídavé

min. vrez kluzu

$$\sigma_{kt} = 0{,}5\,\sigma_{Pt} = 0{,}5 \cdot 340 = 170\ MPa \qquad (k = 1{,}7 \div 2)$$

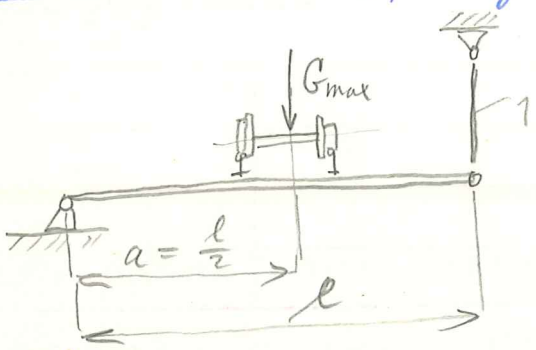
$$\sigma_{Dt\,I} = \frac{\sigma_{kt}}{k} = \frac{170}{(1{,}7 \div 2)} = \underline{(85 \div 100)\ MPa}$$

$$\sigma_{Dt\,II} = 0{,}85\,(85 \div 100) = \underline{(72{,}25 \div 85)\ MPa} \qquad c_{II} = 0{,}85$$

$$\sigma_{Dt\,III} = 0{,}65\,(85 \div 100) = \underline{(55{,}25 \div 65)\ MPa} \qquad c_{III} = 0{,}65$$

4.34 σ_{max}, míjivé, $a = \frac{l}{2}$, 10420 $\quad k = 2$

$c_{II} = 0{,}85$



$\sigma_{Dt} = ?$

$\sigma_{Pt} = 420\ MPa$

$$\sigma_{Dt\,I} = 0{,}6 \frac{\sigma_{Pt}}{k} = 0{,}6 \frac{420}{2} = 126\ MPa$$

$$\underline{\sigma_{Dt\,II}} = c_{II}\,\sigma_{Dt\,I} = 0{,}85 \cdot 126 = \underline{107{,}1\ MPa}$$

4.35 Urči σ_{Dt}, σ_{Dd}, τ_{Ds} pro šedou litinu 42 24 28 míjivé zatížení

$\sigma_{Pt} = 280\ MPa$; $c_{II} = 0,75$; $k_2 = 5$

$\sigma_{Dt\,II} = c_{II} \frac{\sigma_{Pt}}{k_2} = 0,75 \frac{280}{5} = 42\ MPa$ $\quad \sigma_{Dt} \doteq \tau_{Ds}$

$\tau_{Ds\,II} = 42\ MPa$ $\quad \sigma_{Pd} \doteq 3\sigma_{Pt}$

$\sigma_{Dd\,II} = c_{II} \frac{3\sigma_{Pt}}{k_2} = 0,75 \cdot \frac{3 \cdot 280}{5} = 126\ MPa$

4.36 11 425 .... $\sigma_t = 127,6\ MPa$ v tahu — míjivé zatěž.

$k = ?$

$c_{II} = 0,85$; $\sigma_{Pt} = 420\ MPa$

$\sigma_{Kt} = 0,6\,\sigma_{Pt} = 0,6 \cdot 420 = 252\ MPa$

$\sigma_{Dt\,II} = c_{II} \frac{\sigma_{Kt}}{k} \Rightarrow k = c_{II} \frac{\sigma_{Kt}}{\sigma_t} = 0,85 \cdot \frac{252}{127,6} = 1,678$ $\quad \sigma_{Dt} = \sigma_t$

$k \doteq 1,7$

4.37 Tyč Φ $a = ?$ $F = 100\,000\ N$; $\sigma_{Dt} = 110\ MPa$

$S = \frac{F}{\sigma_{Dt}} = \frac{100\,000}{110} = 909\ mm^2$

$S = a^2 \Rightarrow a = \sqrt{S} = \sqrt{909} = 30,2\ mm$

podle ČSN volíme tyč $32 \times 32\ mm$

4.38 Litina 42 24 12 na tlak $F = 300\ kN$; $k = 5$

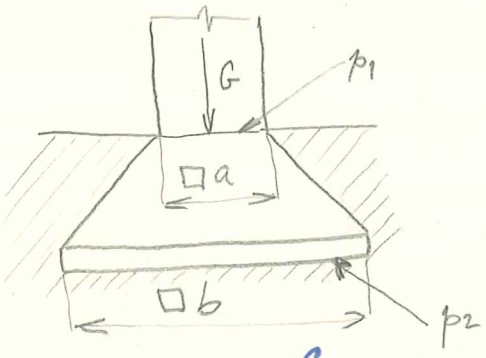
tyč Φ $d = ?$ $\quad$ v tlaku $\sigma_{Pd} = 353\ MPa$

$\sigma_{Dd} = \frac{\sigma_{Pd}}{k} = \frac{353}{5} = 70,6\ MPa$

$S = \frac{F}{\sigma_{Dd}} = \frac{300 \cdot 10^3}{70,6} = 4249,3\ mm$

$d = \sqrt{\frac{4S}{\pi}} = \sqrt{\frac{4 \cdot 4249,3}{\pi}} = 73,5\ mm$

4.39



$G = 200\ kN$

$a = ?$ ⌀ $p_{D1} = 1{,}2\ MPa$

$b = ?$ ⌀ $p_{D2} = 0{,}2\ MPa$

Tíhu základu zanedbat.

$$p = \frac{G}{S_a} \Rightarrow S_a = \frac{G}{p}$$

$$S_a = a^2 = \frac{G}{p_1} = \frac{200\,000}{1{,}2} = 166\,666\ mm^2$$

$$a = \sqrt{166\,666} = \underline{408\ mm}$$

$$S_b = b^2 = \frac{G}{p_2} = \frac{200\,000}{0{,}2} = 1\,000\,000\ mm^2$$

$$b = \sqrt{1\,000\,000} = \underline{1000\ mm}$$

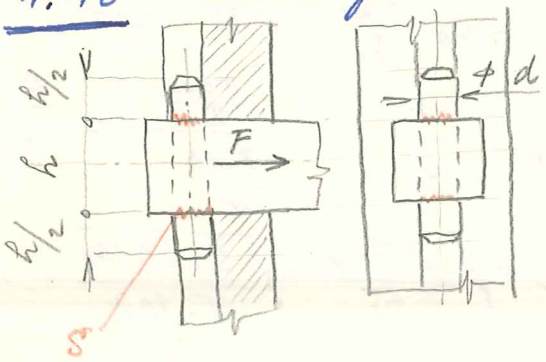
4.40

Táhlo je zatěžováno osovou silou $F = 8000\ N$

Kolík 11500, $k_1 = 2$

$d = ?$

$h = ?$ kolík $p_D = 100\ MPa$



$$\sigma_{Pt} = 500\ MPa \rightarrow \sigma_{Dt} = \frac{\sigma_{Pt} \cdot 0{,}6}{k_1} = \frac{500 \cdot 0{,}6}{2} = 150\ MPa$$

$$\tau_{Ds} \doteq 0{,}6\,\sigma_{Dt} = 0{,}6 \cdot 150 = 90\ MPa$$

$$\tau_{Ds} \leq \frac{F}{2S} = \frac{F}{2\frac{\pi d^2}{4}} = \frac{4F}{2\pi d^2} = \frac{2F}{\pi d^2}$$

$$d = \sqrt{\frac{2F}{\pi \tau_{Ds}}} = \sqrt{\frac{2 \cdot 8000}{\pi \cdot 90}} \geq 7{,}52\ mm \quad \text{volíme } \underline{d = 8\ mm}$$

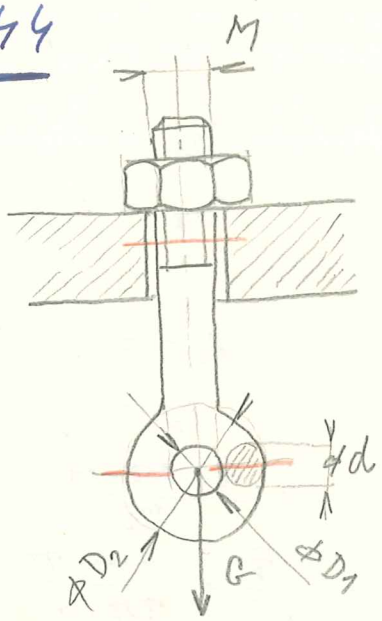
$S_1 = d \cdot \frac{h}{2} \cdot 2$

$S_1 = d \cdot l$

$$p_D = \frac{F}{S_1} = \frac{F}{d \cdot l}$$

$$h = \frac{F}{p_D\, d} = \frac{8000}{100 \cdot 8} = \underline{10\ mm}$$

4.44



$G = 5000\,N$

11341

$k_1 = 2$

$D_1 = 12\,mm$

$d = \frac{1}{2}(D_2 - D_1)$

$M = ?$

$d = ?$

$\sigma_{Pt} = 340\,MPa$

$\sigma_{kt} = 0{,}6\,\sigma_{Pt}$

$\sigma_{Dt} = \frac{\sigma_{kt}}{k_1} = \frac{0{,}6\,\sigma_{Pt}}{k_1} = \frac{0{,}6 \cdot 340}{2} = 102\,MPa$

$S_1 = \frac{F}{\sigma_{Dt}} = \frac{5000}{102} \doteq 50\,mm^2$

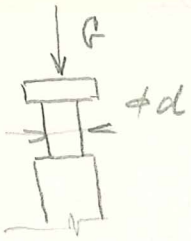
$d_1 = \sqrt{\frac{4S_1}{\pi}} = \sqrt{\frac{4 \cdot 50}{\pi}} = 7{,}98\,mm$ z tab. M10

$\sigma_{Dt} = \frac{F}{2S_2} \Rightarrow S_2 = \frac{F}{2\sigma_{Dt}} = \frac{5000}{2 \cdot 102} = 24{,}5\,mm^2$

$d = \sqrt{\frac{4S_2}{\pi}} = \sqrt{\frac{4 \cdot 24{,}5}{\pi}} = 5{,}58 \doteq 5{,}6\,mm$

$D_2 = D_1 + 2d = 12 + 11{,}2 = 23{,}2\,mm$

4.45



Min. ϕ slonku hevera $d = ?$

$G_{max} = 50\,kN$ 11500

$k_1 = 2$

$G = 0$; G_{max} ⋯ c_{II}

$\sigma_{DII} = c_{II}\frac{\sigma_{kt}}{k_1} = c_{II}\frac{0{,}6\,\sigma_{Pt}}{k_1} = \frac{0{,}6 \cdot 500}{2} \cdot 0{,}85 =$

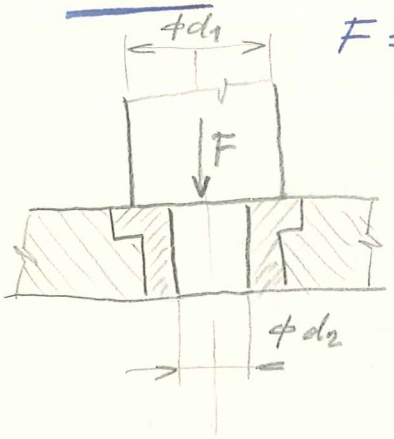
$= 127\,MPa$

$d = \sqrt{\frac{4F}{\pi\,\sigma_{DII}}} = \sqrt{\frac{4 \cdot 50000}{\pi \cdot 127}} = 22{,}39\,mm$

volíme $d = 23\,mm$

4.48

Nitkalený čep zatížený příčnou posuvnou silou $F = 10\,000$ N je uložen v bronzové pánvi; malé otlačení. Navrhněte $d_1 = ?$



$d_2 = 40$ mm

$\sigma_d = ?$ $\quad (p_D = 5{,}8$ MPa)

$$p_D = \frac{F}{S} = \frac{F}{\frac{\pi}{4}(d_1^2 - d_2^2)}$$

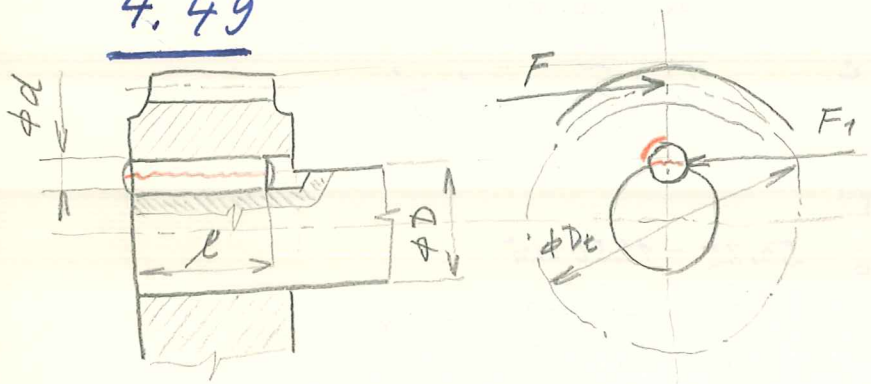
$$p_D \pi (d_1^2 - d_2^2) = 4F$$

$$d_1 = \sqrt{\frac{4F}{p_D \pi} + d_2^2} = \sqrt{\frac{4 \cdot 10\,000}{5{,}8 \cdot \pi} + 40^2}$$

$d_1 = 61{,}6$ mm volíme $\underline{d_1 = 62 \text{ mm}}$

$$\sigma_d = \frac{F}{S_1} = \frac{10\,000}{\frac{\pi d_1^2}{4}} = \frac{4 \cdot 10\,000}{\pi \cdot 62^2} = \underline{3{,}31 \text{ MPa}}$$

4.49



$D_t = 100$ mm; $F = 10\,000$ N

$D = 50$ mm

$d = 10$ mm ... 11500

$l = 70$ mm

$k = 3$

$p_D = 100$ MPa

a) $M_k = ?$

b) $M_{k_1} = ?$ navrženého kolíku

c) $p = ?$

Těžké provozní podmínky

a) $M_k = F \frac{D_t}{2} = 10\,000 \cdot 50 = 500\,000$ Nmm $= \underline{500$ Nm$}$

b) $\tau_s = 0{,}6\,\sigma_{D_t} = 0{,}6 \cdot c_{\parallel} \frac{0{,}6\,\sigma_{Pt}}{k} = 0{,}6 \cdot 0{,}85 \frac{0{,}6 \cdot 500}{3} \doteq 50$ MPa

$F_{1b} = S \cdot \tau_D = l d \tau_{Ds} = 70 \cdot 10 \cdot 50 = 35\,000$ N

$M_{kb} = F_{1b} \cdot \frac{D}{2} = 35\,000 \cdot 25 = 875\,000$ Nmm $= \underline{875$ Nm$}$

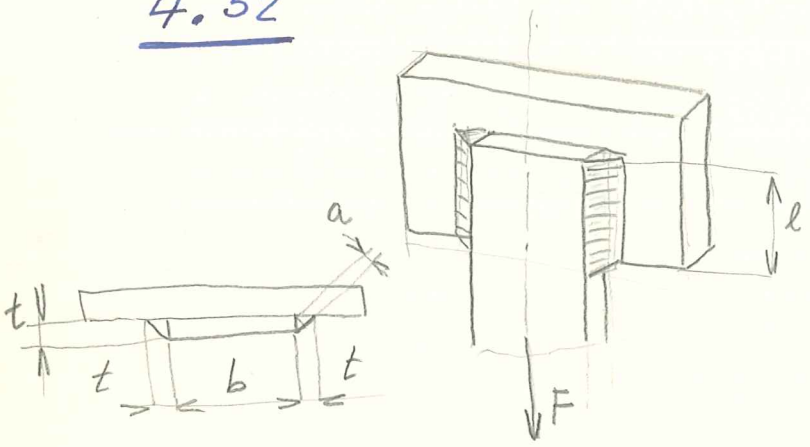
c) $F_1 = \frac{M_k}{\frac{D}{2}} = \frac{500\,000}{25} = 20\,000$ N

$$p = \frac{F_1}{S} = \frac{F_1}{\frac{d}{2} l} = \frac{20\,000}{5 \cdot 70} = \underline{57{,}1 \text{ MPa}}$$

$p_D = 100$ MPa

$p < p_D$ vyhovuje

4.52



$F = 12\,000\ N$

11 340 ; $k = 2$

ČSN 42 5522

$l = ?$

$\tau_{DSV} = 0,65\ \tau_{DZM}$

$\sigma_{Dt} = \frac{0,6\ \sigma_{Pt}}{k} = \frac{0,6 \cdot 340}{2} = 102\ MPa$

$S = \frac{F}{\sigma_{Dt}} = \frac{12\,000}{102} = 117,6\ mm^2$

z tab. nejbližší vyšší $25 \times 5\ mm$

$a = 0,7t = 0,7 \cdot 5 = 3,5\ mm$

$\tau_D = 0,6\ \sigma_{Dt} = 0,6 \cdot 102 = 61,2\ MPa$

$\tau_{DSV} = 0,65\ \tau_{DZM} = 0,65 \cdot 61,2 = 39,8\ MPa$

$\tau_s = \frac{F}{2al} \Rightarrow l = \frac{F}{2a\tau_s} = \frac{12\,000}{2 \cdot 3,5 \cdot 39,8} = 43\ mm$

4.53

$t = 2,5\ mm$ 11 340

$a = 30\ mm$ $F = ?$

$b = 40\ mm$

$\tau_s = 0,8\ \sigma_{Pt} = 0,8 \cdot 340 = 272\ MPa$

$F = \tau_s \cdot S \cdot t = \tau_s\ 2(a+b)t = 272 \cdot 2(30+40) \cdot 2,5 = 95\,200\ N$

$F_1 = \tau_s\ t\ b = 272 \cdot 2,5 \cdot 40 = 27\,200\ N$

$F_2 = \tau_s\ t\ a = 272 \cdot 2,5 \cdot 30 = 20\,400\ N$

4.54

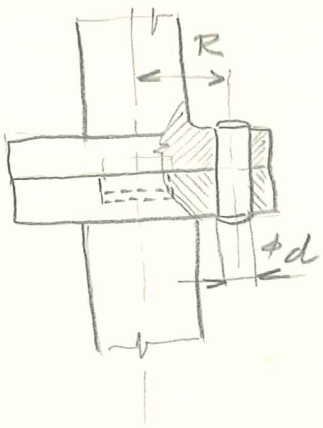
$F = ?$ $d = 20\ mm$; $t = 5\ mm$ 11 370 ... $\sigma_{Pt} = 400\ MPa$

$\tau_s = 0,8 \cdot 400 = 320\ MPa$

$F = S \cdot t \cdot \tau_s = \pi d\ t\ \tau_s = \pi \cdot 20 \cdot 5 \cdot 320 = 100\,480\ N$

$\doteq 100\ kN$

4.55



$P_{max} = 12\,kW$; $\omega = 12\,s^{-1}$

$d = ?$; 11500 ; $R = 40\,mm$

$$M_k = \frac{P}{\omega}\ [Nm] \qquad (\omega \text{ in } s^{-1})$$

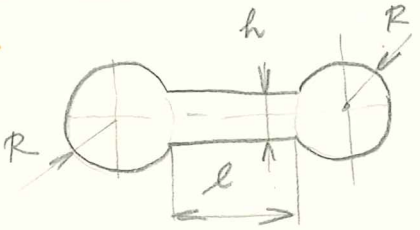
$$M_k = \frac{12000}{12} = 1000\,Nm$$

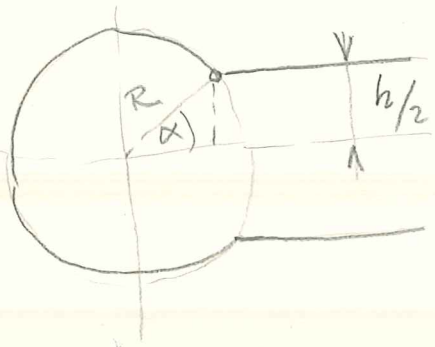
$$F = \frac{M_k}{R} = \frac{1000 \cdot 10^3}{40} = 25\,000\,N$$

$$\tilde{\tau}_S = 0{,}8\,\sigma_{Pt} = 0{,}8 \cdot 500 = 400\,MPa$$

$$\tau_S = \frac{F}{S} = \frac{F}{\frac{\pi d^2}{4}} \Rightarrow d = \sqrt{\frac{4F}{\pi \tilde{\tau}_S}} = \sqrt{\frac{4 \cdot 25\,000}{\pi \cdot 400}} = \underline{\underline{8{,}92\,mm}}$$

4.56





$t = 5\,mm$; $\tilde{\tau}_{Ps} = 300\,MPa$

$l_1 = 100\,mm$; $R = 30\,mm$; $h = 30\,mm$

$$\sin\alpha = \frac{h/2}{R} = \frac{15}{30} \dots \alpha = 30°$$

$$\alpha_1 = 360 - 2\alpha = 300°$$

$$\sigma_1 = \frac{\pi\alpha}{360} \cdot 300 = \frac{\pi \cdot 60}{360} \cdot 300 = 157\,mm$$

$$\sigma_c = 2\sigma_1 + 2l = 2(\sigma_1 + l) = 2(157 + 100) = \underline{\underline{514\,mm}}$$

$$F = \tilde{\tau}_{Ps} \cdot \sigma_c \cdot t = 300 \cdot 514 \cdot 5 = 771\,000 = \underline{\underline{771\,kN}}$$

4.57

$G = 30\,000\,N$

$n = 6$ ČSN 02 43 22

$i = 19$, $d = 2{,}8\,mm$

$l_0 = 25\,m$; $q = 62\,N\,m^{-1}$

$E = 2{,}1 \cdot 10^5\,MPa$

a) $\sigma_{t\,max} = ?$

b) $\Delta l = ?$

$Q = l_0 q = 25 \cdot 62 = 1550\,N$

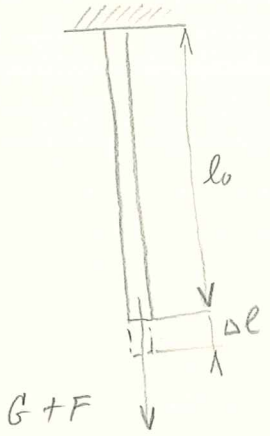
$$\sigma_{t\,max} = \frac{G+Q}{S} = \frac{G+Q}{n i \frac{\pi d^2}{4}} = \frac{30\,000 + 1550}{\frac{\pi \cdot 2{,}8^2}{4} \cdot 6 \cdot 19} = 44{,}9\,MPa$$

$$\sigma = E\varepsilon = E\frac{\Delta l}{l_0}$$

$$\sigma_p = \frac{G + \frac{Q}{2}}{n i \frac{\pi d^2}{4}} = \frac{30\,775}{\frac{\pi \cdot 2{,}8^2}{4} \cdot 6 \cdot 19} = 43{,}86\,MPa$$

$$\Delta l = \frac{\sigma l_0}{E} = \frac{43{,}86 \cdot 25000}{2{,}1 \cdot 10^5} = \underline{\underline{5{,}22\,mm}}$$

4.59



$l_0 = 30\,m$; $d_0 = 40\,mm$

G – vlastní váha

$F = 150 \cdot 10^3\,N$; $\rho = 78 \cdot 10^3\,N\,m^{-3}$

$\Delta l = 18\,mm$

a) $\sigma_{max} = ?$

b) $E = ?$

$$G = V\rho = \frac{\pi d^2}{4} l_0 \rho = \frac{\pi\, 40^2 \cdot 10^{-6}}{4} \cdot 30 \cdot 78 \cdot 10^3 =$$

$$= \pi \cdot 10 \cdot 40 \cdot 30 \cdot 78 \cdot 10^{-3} = \underline{2939\,N}$$

a) $$\sigma_{max} = \frac{F+G}{\frac{\pi d^2}{4}} = \frac{4\,(150\,000 + 2939)}{\pi \cdot 40^2} = \underline{121{,}3\,MPa}$$

b) $$\sigma = \frac{F + \frac{G}{2}}{\frac{\pi d^2}{4}} = \frac{4\,(150\,000 + 1196)}{\pi \cdot 40^2} = \underline{120{,}3\,MPa}$$

$$E = \frac{\sigma l_0}{\Delta l} = \frac{120{,}3 \cdot 30\,000}{18} = 200\,500 = \underline{2{,}005 \cdot 10^5\,MPa}$$

4.69

11 353 $\quad \sigma_{Pt} = 350\ MPa$ $\quad D = 200\ mm$, $p = 5\ MPa$

$\sigma_{kt} = 0{,}6 \cdot 350$

$$\sigma_{Dt} = \frac{\sigma_{kt}}{k} = \frac{0{,}6 \cdot 350}{2} = 105\ MPa$$

$$t \geq \frac{d_p}{2\sigma_{Dt}} = \frac{200 \cdot 5}{2 \cdot 105} = 4{,}76\ mm$$

$$t = 5\ mm$$

4.70

$t = 420\,^\circ C$ 17 255

$d_1 = 160\ mm$

$t = 12\ mm$

$\sigma_{Pt} = 600 \div 750\ MPa$ } tab.

$\sigma_{kt} = 250\ MPa$

$$\sigma_{Dt} = \frac{\sigma_{kt}}{k_1} = \frac{250}{1{,}7} = 147\ MPa$$

$\sigma_{Dt\,400} = 47\ MPa$

$$p_D = \frac{2\sigma_{Dt} \cdot t}{d} = \frac{2 \cdot 47 \cdot 12}{160} = 7{,}05\ MPa$$

4.71

Kotle $D = 200\ mm$ $\quad p_{min} = 0$ $\quad$ až $p_{max} = 10\ MPa$

11 444

$k_1 = 2{,}5$

$\varphi = 0{,}9$

$$\sigma_{D\,II} = c_{II} \frac{0{,}6\sigma_{Pt}}{k_1} \varphi$$

$$= 0{,}85 \frac{0{,}6 \cdot 440}{2{,}5} \cdot 0{,}9 = 80{,}78\ MPa$$

$$t \geq \frac{d_p}{4\sigma_{D\,II}} = \frac{200 \cdot 10}{4 \cdot 80{,}78} = 6{,}18\ mm$$

4.72

11 450 $\quad p = 0{,}4\ MPa$ $\quad D = 500\ mm$

$k_1 = 2$

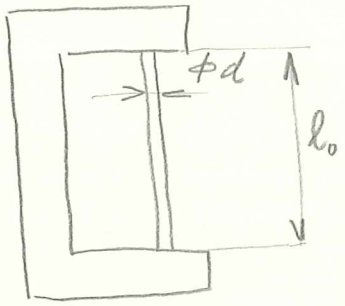
$\varphi = 0{,}9$

$\sigma_{Pt} = 450\ MPa$

$$\sigma_{Dt} = \varphi \frac{\sigma_k}{k_1} = 0{,}9 \frac{0{,}6 \cdot 450}{2} = 121{,}5\ MPa$$

$$t \geq \frac{d_p}{2\sigma_{Dt}} = \frac{500 \cdot 0{,}4}{2 \cdot 121{,}5} = 0{,}823\ mm$$

4.63



$\Delta t = ?$

$\sigma_D = 50,62\ MPa$

$E = 2,05 \cdot 10^5\ MPa$

$\alpha = 12 \cdot 10^{-6}\ K^{-1}$

$\sigma = E \alpha \Delta t$

$$\Delta t = \frac{\sigma}{E\alpha} = \frac{50,62}{2,1 \cdot 10^5 \cdot 12 \cdot 10^{-6}} = 20,4\ °C$$

4.64

$l_0 = 600\ mm \qquad t_1 = 20°C$

$\sigma = ? \qquad t_2 = 100°C$

$\sigma_{kt} = 200\ MPa$

$E = 2,1 \cdot 10^5\ MPa$

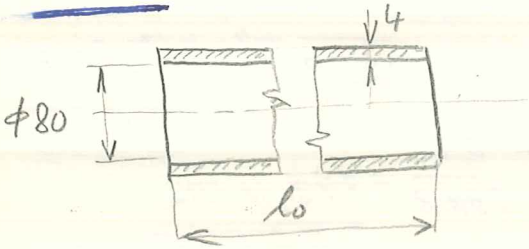
$\alpha = 16,7 \cdot 10^{-6}\ K^{-1}$

$\sigma_d = E \alpha \Delta t = 2,1 \cdot 10^5 \cdot 16,7 \cdot 10^{-6} \cdot 80 =$

$= 280,56\ MPa$

$\sigma_d > \sigma_{kt}$ – prvek se dělá zvonu.

4.65



$l_0 = 30\ m \qquad t_1 = 25°C$

$\Delta l = ? \qquad t_2 = -15°C$

$F = ? \qquad E = 2,1 \cdot 10^5\ MPa$

$\sigma = ? \qquad \alpha = 12 \cdot 10^{-6}\ K^{-1}$

~~Δt~~

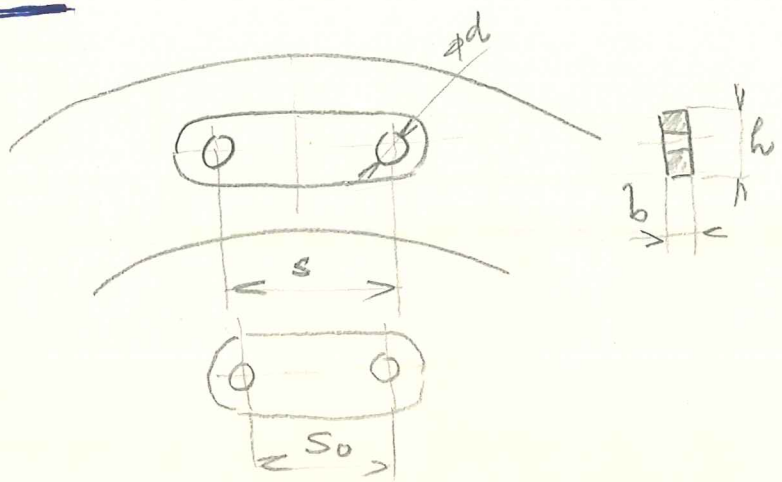
$\sigma = E \alpha \Delta t = 2,1 \cdot 10^5 \cdot 12 \cdot 10^{-6} \cdot 40 =$

$= 100,8\ MPa$

$\Delta l = l_0 \alpha \Delta t = 30\,000 \cdot 12 \cdot 10^{-6} \cdot 40 = 14,4\ mm$

$F = \sigma S = \sigma \pi (d+4) \cdot 4 = 100,8 \cdot \pi \cdot 84 \cdot 4 = 106\,348\ N$

4.66



$d = 80\ mm \qquad s_0 = ?$

$s = 300\ mm$

$\sigma_t = 120\ MPa \qquad F = ?$

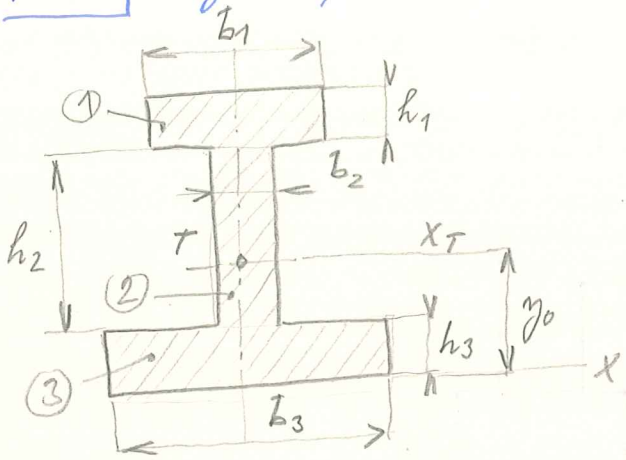
$b = 40\ mm$

$h = 160\ mm$

$E = 2,0 \cdot 10^5\ MPa$

$\tau_{DS} = 98,1\ MPa$

4.76 | Zjisti polohu těžiště $y_0 = ?$



$b_1 = 340\ mm$
$h_1 = h_3 = 100\ mm$
$b_2 = 80\ mm$
$h_2 = 450\ mm$
$b_3 = 480$

$S_1 = b_1 h_1 = 340 \cdot 100 = 3,4 \cdot 10^4\ mm^2$
$S_2 = b_2 h_2 = 80 \cdot 450 = 3,6 \cdot 10^4\ mm^2$
$S_3 = b_3 h_3 = 480 \cdot 100 = 4,8 \cdot 10^4\ mm^2$
$S = 11,8 \cdot 10^4\ mm^2$

Stat. moment

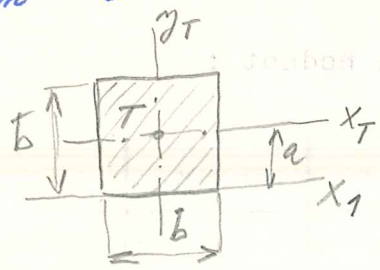
$$S_1 \cdot y_1 + S_2 \cdot y_2 + S_3 y_3 = S \cdot y_0$$

$$y_0 = \frac{S_1 y_1 + S_2 y_2 + S_3 y_3}{S} = \frac{3,4 \cdot 6 + 3,6 \cdot 3,25 + 4,8 \cdot 0,5}{11,8} =$$

$$= \frac{20,4 + 11,7 + 2,4}{11,8} = 2,92\ dm = 292\ mm$$

4.77 | čtverec $b = 120\ mm$ $J = ?$

a) k ose x_T
b) k ose x_1
c) k ose y_T



a) $J_x = \frac{b^2}{12} = \frac{120^4}{12} = 17280000 = 1728 \cdot 10^4\ mm^4$

b) $J_{x_1} = J_x + S a^2 = 1728 \cdot 10^4 + 120^2 \cdot 60^2 = 1728 \cdot 10^4 + 5184 \cdot 10^4 = 6912 \cdot 10^4\ mm^2$

c) $J_x = J_y = 1728 \cdot 10^4\ mm^2$ $\quad \Sigma \Delta S y^2 = \Sigma \Delta S x^2 \dots x = y$

4.78 | $\phi 17$ J_x, J_p, j'

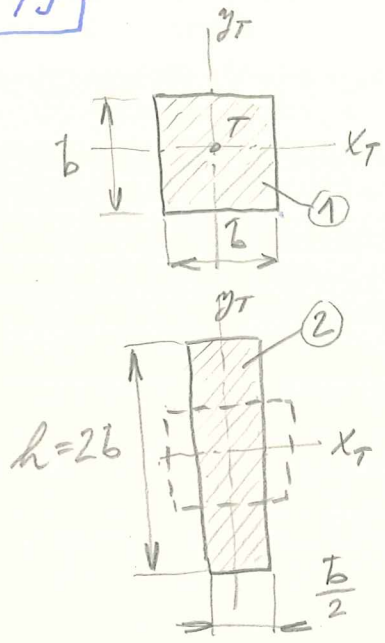
$$J_p = \frac{\pi}{32} d^4 = \frac{\pi}{32} 17^4 = 8199,6\ mm^4$$

$$J_x = \frac{J_p}{2} = \frac{8199,6}{2} = 4099,8\ mm^4$$

$$j' = \sqrt{\frac{J_x}{S}} = \sqrt{\frac{4099,8}{226,9}} = 4,25\ mm$$

$$S = \frac{\pi d^2}{4} = \frac{\pi \cdot 17^2}{4} = 226,9\ mm^2$$

4.79



S □ a ▭ jsou stejná

□ $b = 120\,mm$

▭ $\frac{b}{2}$; $h = 2b$

a) $J_x = ?$ $J_y = ?$

b) poměr momentů ?

---

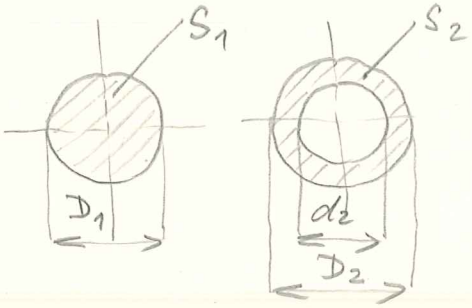
a) $J_{x\square} = J_{y\square} = \frac{b^4}{12} = \frac{120^4}{12} = \underline{1728 \cdot 10^4\,mm^4}$

$J_{x\sqsupset} = \frac{\frac{b}{2} h^3}{12} = \frac{60 \cdot 240^3}{12} = \underline{6912 \cdot 10^4\,mm^4}$

$J_{y\sqsupset} = \frac{2b \cdot (\frac{b}{2})^3}{12} = \frac{240 \cdot 60^3}{12} = \underline{432 \cdot 10^4\,mm^4}$

b) $p = \frac{J_{x\square}}{J_{x\sqsupset}} = \frac{1728 \cdot 10^4}{6912 \cdot 10^4} = 0{,}25 = \underline{\underline{\frac{1}{4}}}$

4.80



$S_1 = S_2 = S$

$S = 12{,}56 \cdot 10^2\,mm^2$

$D_1 = 40\,mm$

$D_2 = 50\,mm$

$d_2 = 30\,mm$

$J_p = ?$

$J_x = ?$

$p = ?$

$J_{p\bullet} = \frac{\pi}{32} D_1^4 = \frac{\pi}{32} 40^4 = \underline{\underline{25{,}12 \cdot 10^4\,mm^4}}$

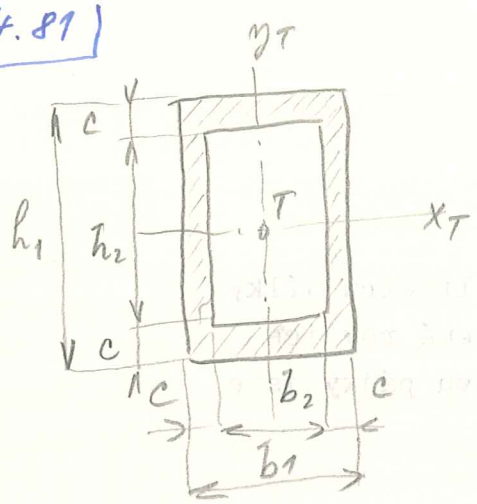
$J_x = J_y = \frac{J_{p\bullet}}{2} = \underline{\underline{12{,}56 \cdot 10^4\,mm^4}}$

$J_{p\circledcirc} = \frac{\pi}{32}(D_2^4 - d_2^4) = \frac{\pi}{32}(50^4 - 30^4) = \underline{\underline{53{,}3 \cdot 10^4\,mm^4}}$

$J_{x\circledcirc} = \frac{J_{p\circledcirc}}{2} = \underline{\underline{26{,}65 \cdot 10^4\,mm^4}}$

$p = \frac{J_{p\bullet}}{J_{p\circledcirc}} = \frac{25{,}12 \cdot 10^4}{53{,}3 \cdot 10^4} = 0{,}471 \ldots \frac{1}{2{,}12}$

4.81



$b_1 = 100\,mm$

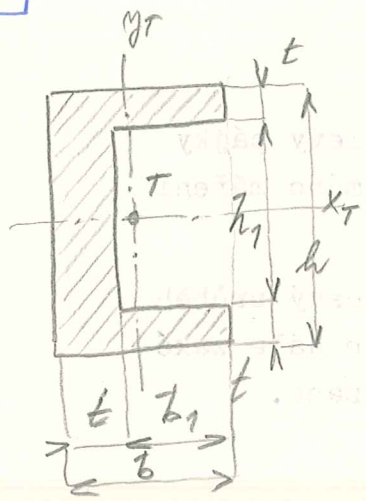
$h_1 = 200\,mm$

$c = 20\,mm$

$$J_x = \frac{b_1 h_1^3 - b_2 h_2^3}{12} = \frac{100 \cdot 200^3 - 60 \cdot 160^3}{12} = 5542 \cdot 10^5\ mm^4$$

$$J_y = \frac{b_1^3 h_1 - b_2^3 h}{12} = \frac{100^3 \cdot 200 - 60^3 \cdot 160}{12} = 1654{,}4 \cdot 10^5\ mm^4$$

4.82



$b = 50\,mm$

$h = 100\,mm$

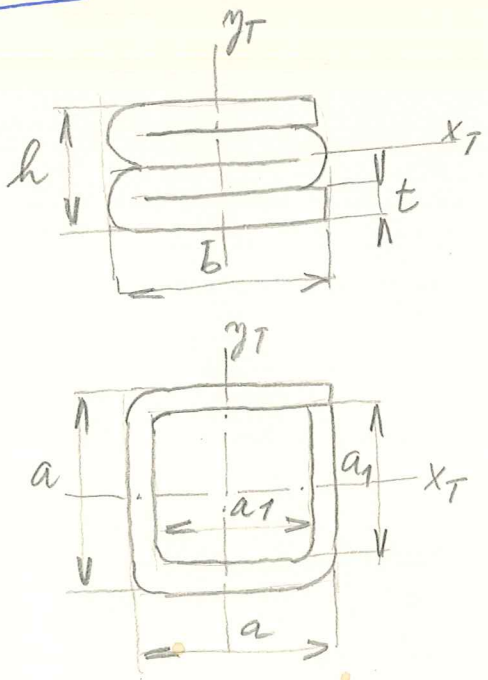
$b_1 = 42\,mm$

$h_1 = 84\,mm$

$$J_x = \frac{b h^3 - b_1 h_1^3}{12} = \frac{50 \cdot 100^3 - 42 \cdot 84^3}{12} =$$

$$= 2092 \cdot 10^4\ mm^4$$

4.83



$t = 2\,mm$ $\quad a = 42\,mm$

$l = 160\,mm$ $\quad a_1 = 38\,mm$

$b = 40\,mm$

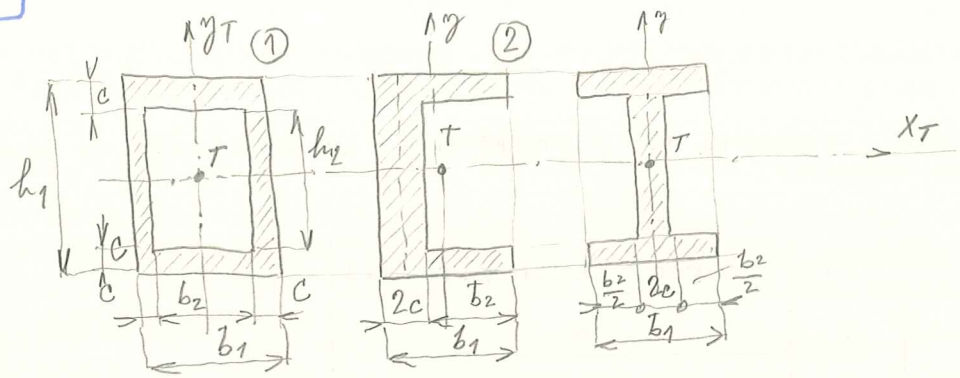
$h = 4t$

$$J_{x_1} = \frac{b h^3}{12} = \frac{40 \cdot 8^3}{12} = 1706{,}6\ mm^4$$

$$J_{x_2} = \frac{a^4 - a_1^4}{12} = \frac{42^4 - 38^4}{12} = 85546{,}6\ mm^4$$

$$\beta = \frac{J_{x_1}}{J_{x_2}} = \frac{1706{,}6}{85546{,}6} \doteq 0{,}02 \cdots \frac{1}{50}$$

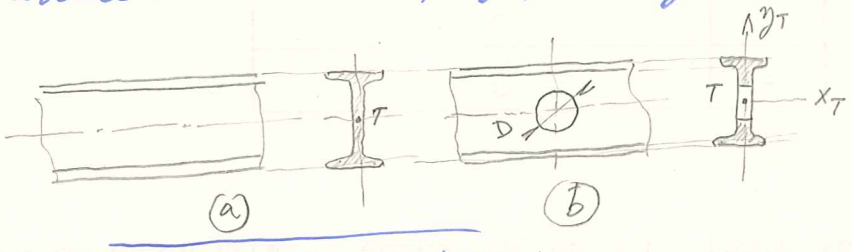
4.84 Které ze tří momentů 1, 2, 3 k ose x_T a y_T jsou stejné?



K kvadr. momentu k ose x_T je u 1, 2, 3 stejný

y_T není ani u jednoho stejný

4.85 Zjistěte kvadr. moment profilu I 30 k ose x_T a dále kvadr. moment, když bude vyvrtán otvor $\phi D = 160$ mm



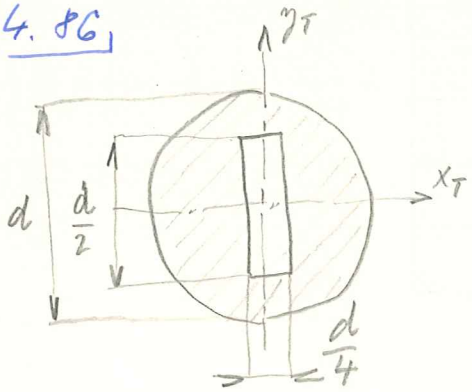
ČSN 42 5550 ST 403

a) $J_x = 9800 \cdot 10^4 \text{ mm}^4$

b) $J_{x_D} = J_x - \frac{bh^3}{12} = 9800 \cdot 10^4 - \frac{10{,}8 \cdot 160^3}{12} = 10^4 (9800 - 368{,}64) =$

$= 9431{,}36 \cdot 10^4 \text{ mm}^4$

4.86



$J_x = ?$ $\quad d = 100$ mm

$J_y = ?$

$J_{xk} = \frac{\pi d^4}{64} = \frac{\pi}{64} (10^2)^4 = 0{,}049 \cdot 10^8 = 490 \cdot 10^4 \text{ mm}^4$

$J_{x_0} = \frac{bh^3}{12} = \frac{25 \cdot 50^3}{12} = 26{,}04 \cdot 10^4$

$J_x = J_{xk} - J_{x_0} = 10^4 (490 - 26{,}04) = 463{,}96 \cdot 10^4 \text{ mm}^4$

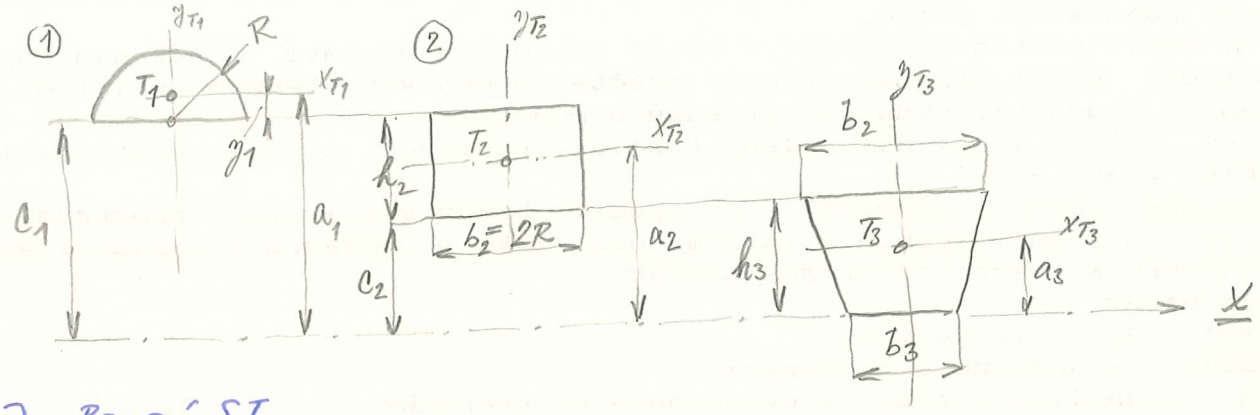
$J_{y_0} = \frac{hb^3}{12} = \frac{50 \cdot 25^3}{12} = 6{,}51 \cdot 10^4 \text{ mm}^4$

$J_y = J_{yk} - J_{y_0} = 10^4 (490 - 6{,}51) = 483{,}49 \cdot 10^4 \text{ mm}^4$ $\quad J_{yk} = J_{xk}$

4.87 Vypočti kvadr. moment plochy 1, 2, 3 k mimotěžišťové ose x

$R = 20\,mm$, $c_1 = 50\,mm$, $b_2 = 40\,mm$, $h_2 = 30\,mm$, $c_2 = 20\,mm$; $b_3 = 30\,mm$

$h_3 = c_2 = 20\,mm$



① Pomocí ST

$y_1 \doteq 0{,}42R = 0{,}42 \cdot 20 = 8{,}4\,mm$

$a_1 = c_1 + y_1 = 50 + 8{,}4 = 58{,}4\,mm$

$J_{x_1} = \frac{1}{2}\frac{\pi}{64}d^4 = \frac{1}{2}\frac{\pi \cdot 40^2}{64} = 62\,800\,mm^4$

$S_1 = \frac{1}{2}\pi\frac{d^2}{4} = \frac{1}{8}\pi \cdot 40^2 = 628\,mm^2$

$J_{xT_1} = J_{x_1} - S_1 y_1^2 = 62\,800 - 628 \cdot 8{,}4^2 = 18\,488\,mm^4$

$J_x = J_{xT_1} + S a_1^2 = 18\,488 + 628 \cdot 58{,}4^2 = \underline{216{,}03 \cdot 10^4\,mm^4}$

②

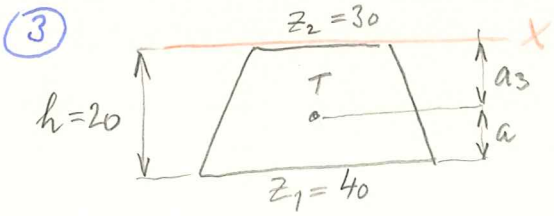
$J_{xT_2} = \frac{b_2 h_2^3}{12} = \frac{40 \cdot 30^3}{12} = 90\,000\,mm^4$

$S_2 = b_2 h_2 = 40 \cdot 30 = 1200\,mm^4$

$a_2 = c_2 + \frac{h_2}{2} = 20 + 15 = 35\,mm$

$J_x = J_{xT_2} + S_2 a_2^2 = 90000 + 1200 \cdot 35^2 = 1560\,000 = \underline{156 \cdot 10^4\,mm^4}$

③



$z_1 + z_2$

$a = \frac{h}{3}\frac{z_1 + 2z_2}{z_1 + z_2} = \frac{20\,(40 + 2 \cdot 30)}{3\,(40 + 30)} = 9{,}523\,mm$

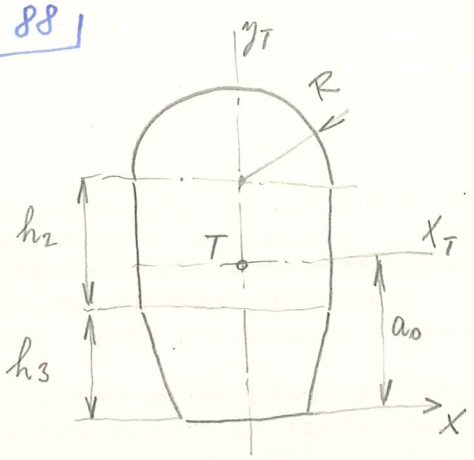
$a_3 = 20 - 9{,}523 = \underline{10{,}477\,mm}$

$S_3 = h\frac{z_1 + z_2}{2} = 20\frac{40 + 30}{2} = \underline{700\,mm^2}$

$J_{xT} = \frac{h^3}{36}\frac{z_1^2 + 4z_1z_2 + z_2^2}{z_1 + z_2} = \frac{20^3}{36}\frac{40^2 + 4 \cdot 40 \cdot 30 + 30^2}{40 + 30} = 23\,171\,mm^4$

$J_x = J_{xT} + S_3 a_3^2 = 23\,171 + 700 \cdot 10{,}477^2 = \underline{10 \cdot 10^4\,mm^4}$

4.88



Údaje jsou z př. 4.87

$J_{xT} = ?$

$J_x = J_{xT} + S a_0^2$

$J_{xT} = J_x - S a_0^2$

$a_0 = ?$

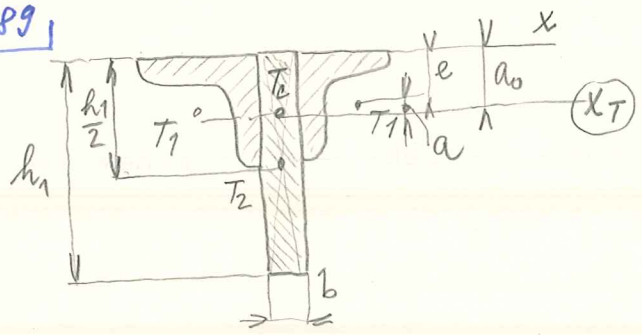
$S = S_1 + S_2 + S_3 = 628 + 1200 + 700 = 2528\ mm^2$

$$a_0 = \frac{S_1 a_1 + S_2 a_2 + S_3 a_3}{S} = \frac{628 \cdot 58{,}4 + 1200 \cdot 35 + 700 \cdot 10{,}47}{2528} = 34{,}02\ mm$$

$J_{xc} = J_{x_1} + J_{x_2} + J_{x_3} = 10^4 (216{,}03 + 156 + 10) = 382{,}03 \cdot 10^4\ mm^4$

$J_{xT} = J_{xc} - S a_0^2 = 382{,}03 \cdot 10^4 - 2528 \cdot 34^2 = 89{,}83 \cdot 10^4\ mm^4$

4.89



L 50×50×6 ČSN 42 5541

$b_1 = 12\ mm$

$h_1 = 100\ mm$

$J_{xT} = ?$

$J_{yT} = ?$

ST 401

$e = 14{,}4\ mm$

$S_1 = 569\ mm^2$

$J_x = 12{,}88 \cdot 10^4\ mm^4$

$S_2 = b \cdot h = 12 \cdot 100 = 1200\ mm^2$

$$a_0 = \frac{2 \cdot S_1 e + S_2 \cdot \frac{h}{2}}{S} = \frac{2 \cdot 569 \cdot 14{,}4 + 1200 \cdot 50}{2338} = 32{,}67\ mm$$

$S = 2S_1 + S_2 = 2 \cdot 569 + 1200 = 2338\ mm^2$

$a = a_0 - e = 32{,}67 - 14{,}4 = 18{,}27\ mm$

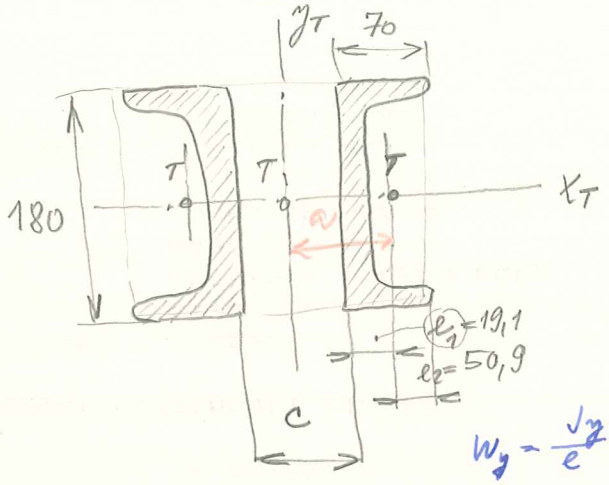
$J_{xT1} = (J_{x_1} + S_1 a^2) 2 = 2(12{,}88 \cdot 10^4 + 569 \cdot 18{,}27^2) = 2(12{,}88 \cdot 10^4 + 18{,}99 \cdot 10^4) = 63{,}74 \cdot 10^4\ mm^4$

$J_{xT2} = \frac{b h^3}{12} + S_2 \left(\frac{h}{2} - a_0\right)^2 = \frac{12 \cdot 100^3}{12} + 1200 \cdot 17{,}33^2 = 136{,}08 \cdot 10^4\ mm^4$

$J_{xT} = J_{xT1} + J_{xT2} = 63{,}74 \cdot 10^4 + 136{,}08 \cdot 10^4 = 199{,}7 \cdot 10^4\ mm^4$

$J_x = J_{xT} + S a_0^2 = 199{,}7 \cdot 10^4 + 2338 \cdot 32{,}67^2 = 449{,}2 \cdot 10^4\ mm^4$

4.90 | Vjaké vodálenosti mezi gL 2 profily U 18 ČSN 42 5570 aby $J_x = J_y$? $c = ?$



ST 404

$J_{xT} = 1350 \cdot 10^4 \, mm^4$

$S_u = 2800 \, mm^2$

$e_1 = ?$

$J_{yT} = 114 \cdot 10^4 \, mm^4$

$W_y = 22,4 \cdot 10^3 \, mm^3$

$W_y = \frac{J_y}{e} \Rightarrow e_2 = \frac{J_y}{W_y} = \frac{114 \cdot 10^4}{22,4 \cdot 10^3} = 50,9 \, mm$

$e_1 = b - e_2 = 70 - 50,9 = 19,1 \, mm$

$J_{xTc} = 2 J_{xT} = 2 \cdot 1350 \cdot 10^4 = 2700 \cdot 10^4 \, mm^4$

$J_{yTc} = 2 (J_y + S a^2)$ $\quad a = \frac{c}{2} + e_1$

$J_{xTc} = J_{yTc} = 2 (J_y + S a^2)$

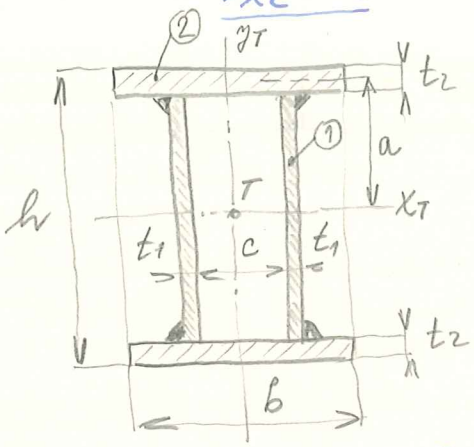
$\frac{J_{xTc}}{2} = J_y + S a^2$

$\frac{J_{xTc}}{2} - J_y = S a^2$

$a = \sqrt{\frac{\frac{J_{xTc}}{2} - J_y}{S}} = \sqrt{\frac{(\frac{2700}{2} - 114) 10^4}{2800}} = 10^2 \cdot 0,664 = 66,4 \, mm$

$c = 2 (a - e_1) = 2 (66,4 - 19,1) = 94,6 \, mm$

4.91] Stanovte J_{xc} = ? složeného profilu $t_1 = 10\,mm$; $t_2 = 20\,mm$, $b = 300$, $h = 400$



$$h_1 = h - 2t_2 = 400 - 2 \cdot 20 = 360\,mm$$

$$a = \frac{h}{2} - \frac{t_2}{2} = 200 - 10 = 190\,mm$$

① $J_{xT_1} = \frac{2 t_1 h_1^3}{12} = \frac{20 \cdot 360^2}{12} = 7776 \cdot 10^4\,mm^4$

② $J_{xT_2} = 2\,(J_x + S a^2)$

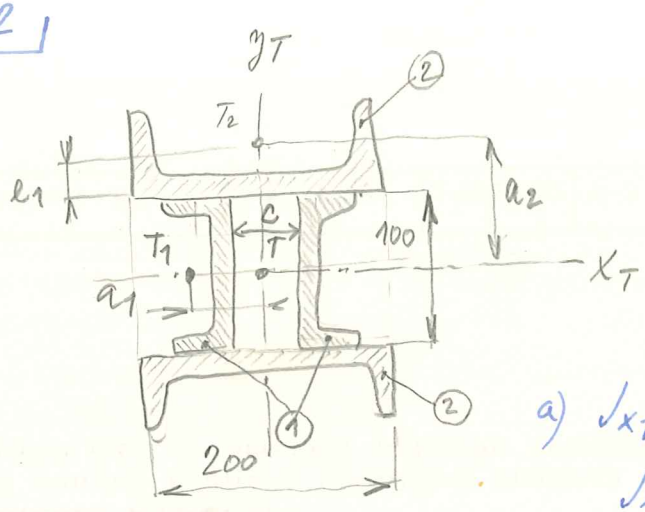
$$J_x = \frac{b \cdot t_2^3}{12} = \frac{300 \cdot 20^3}{12} = 20 \cdot 10^4\,mm^4$$

$$J_{xT_2} = 2\,(20 \cdot 10^4 + 6000 \cdot 190^2) = 2\,(20 + 21660) \cdot 10^4 = 43360 \cdot 10^4\,mm^4$$

$S = b \cdot t_2 = 300 \cdot 20 = 6000\,mm^2$

$$J_{xc} = J_{xT_1} + J_{xT_2} = (7776 + 43360) \cdot 10^4 = 51136 \cdot 10^4\,mm^4$$

4.92



$c = 40\,mm$

① .... U 100
② .... U 200

ST 404

a) $J_{xT} = ?$
b) $J_{yT} = ?$

a) $J_{xT} = 2J_{x_1} + 2(J_{y_2} + S_2 a_2^2)$

$J_{x_1} = 206 \cdot 10^4\,mm^4$
$J_{y_2} = 148 \cdot 10^4\,mm^4$
$S_2 = 3220\,mm^2$

$a_2 = 50 + e_1 = 50 + 20{,}2 = 70{,}2\,mm$

$e_1 = b_2 - e_2 = 75 - 54{,}8 = 20{,}2\,mm$

$e_2 = \frac{J_{y_2}}{W_{y_2}} = \frac{148}{27} = 5{,}49\,cm = 54{,}8\,mm$

$J_{xT} = 2J_{x_1} + 2(J_{y_2} + S_2 a_2^2) = 2 \cdot 206 \cdot 10^4 + 2(148 \cdot 10^4 + 3220 \cdot 70{,}2^2)$

$= 412 \cdot 10^4 + 2(148 \cdot 10^4 + 1586{,}8 \cdot 10^4) = 10^4(412 + 3468) = \underline{3880 \cdot 10^4\,mm^4}$

b) $J_{yT} = 2J_{x_2} + 2(J_{y_1} + S_1 a_1^2)$

$J_{x_2} = 1910 \cdot 10^4\,mm^4$
$J_{y_1} = 29{,}3 \cdot 10^4\,mm^4$
$S_1 = 1350\,mm^2$

$a_1 = \frac{c}{2} + e_1 = 20 + 15{,}5 = 35{,}5\,mm$

$e_2 = \frac{J_{y_1}}{W_{y_2}} = \frac{29{,}3}{8{,}49} = 3{,}45\,cm = 34{,}5\,mm$

$e_1 = b_1 - e_2 = 50 - 34{,}5 = 15{,}5\,mm$

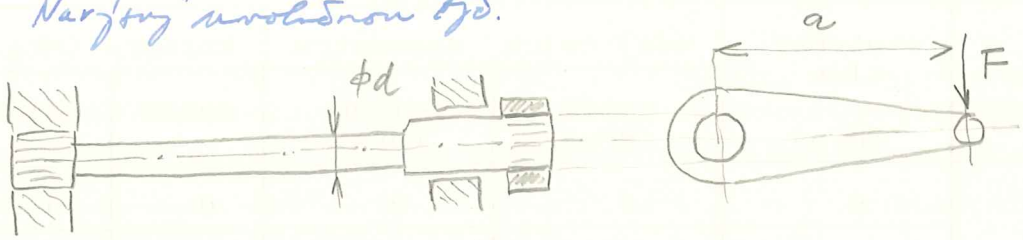
$J_{yT} = 2 \cdot 1910 \cdot 10^4 + 2(29{,}3 \cdot 10^4 + 1350 \cdot 35{,}5^2) =$

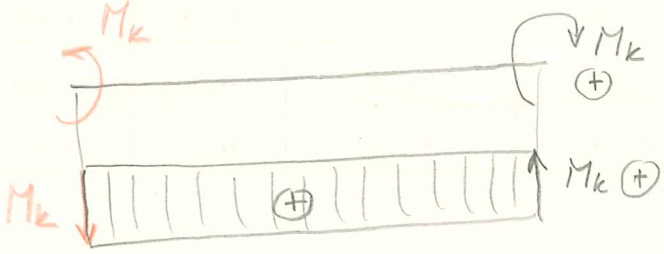
$= 3820 \cdot 10^4 + 2(29{,}3 \cdot 10^4 + 170{,}13 \cdot 10^4) =$

$= 10^4(3820 + 398{,}86) = \underline{4218{,}8 \cdot 10^4\,mm^4}$

KRUT

4.93 $F = ?$ na konci páky $a = 200\,mm$ hřídel je zatížena $M_k = 200\,Nm$
Navrhněte navrhovanou hřídel.

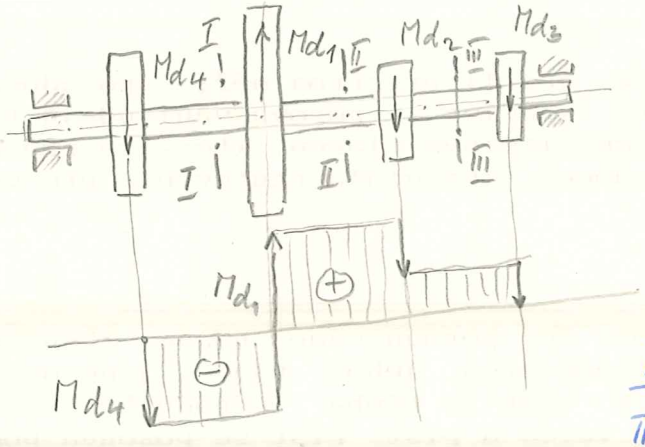




$$M_k = F.a$$

$$F = \frac{M_k}{a} = \frac{200.10^3}{200} = 1000\,N$$

4.94 d = konst.

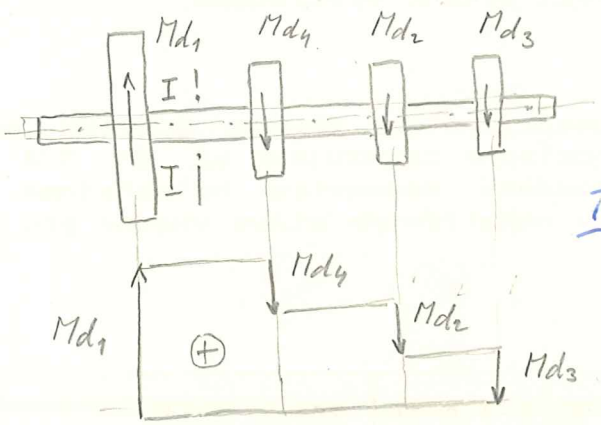


$M_{d1} = 640\,Nm$
$M_{d2} = -210\,Nm$
$M_{d3} = -170\,Nm$
$M_{d4} = -260\,Nm$

I ... $M_{kI} = -260\,Nm$
II ... $M_{kII} = 640 - 260 = +380\,Nm$
III ... $M_{kIII} = -170\,Nm$

Nejvíce je namáhán průřez II

4.95 Hřídel jako v 4.94 jen kolo M_{d1} bude zavěšeno za M_{d4}



I .... $M_{kI} = 640\,Nm$

Toto rozdělení je nevýhodnější

$$M_{kmax} = \frac{640}{380} = 1{,}68$$

To je 1,68 větší než M_{kII}

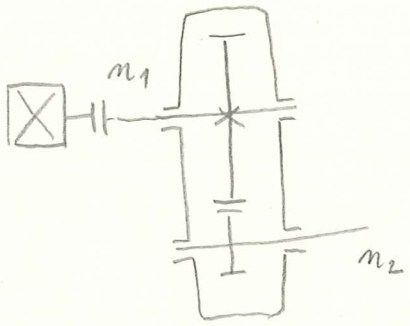
4.96 | Kotouč $D = 300\,mm$; $P = ?$ na hřídeli
když $\omega = 30\,rad\,s^{-1}$ a přenášená obvod. síla $F = 600\,N$

$$M_k = F\frac{D}{2} = 600 \cdot \frac{300}{2} = 90\,000\,Nmm = 90\,Nm$$

$$M_k = \frac{P}{\omega} \Rightarrow \quad P = \omega M_k = 30 \cdot 90 = 2700\,W = \underline{2{,}7\,kW}$$

4.97 | Převodovka $P = 10\,kW$; $n_1 = 12\,s^{-1}$

$M_{k1} = ?$, $M_{k2} = ?$ $\quad i = 1:3$ ztráty zanedbáme



$$i = \frac{n_1}{n_2} = \frac{1}{3} \quad \ldots \text{převod dopředu}$$

$$\omega_1 = 2\pi n_1 = 2\pi \cdot 12 = 75{,}39\,rad\,s^{-1}$$

$$M_{k1} = \frac{P}{\omega_1} = \frac{10 \cdot 10^3}{75{,}39} = \underline{132{,}64\,Nm}$$

$$M_{k2} = \frac{P}{3\omega_1} = \frac{10 \cdot 10^3}{3 \cdot 75{,}39} = \underline{44{,}21\,Nm}$$

4.98 | Hřídel z oceli 11343 střídavě ohýbaný

$\tau_{Dk} = ?$

$k = 1{,}6 \div 1{,}7$

$\sigma_{Pt} = 340\,MPa$

$c_{III} = 0{,}65$

$\sigma_{kt} = 0{,}6 \cdot 340 = 204\,MPa$

$$\sigma_{Dt_{III}} = c_{III}\frac{\sigma_{kt}}{k} = 0{,}65\,\frac{204}{1{,}7} = 78\,MPa$$

$$\tau_{Dk_{III}} = (0{,}6 \div 0{,}65)\,\sigma_{Dt} = 0{,}6 \cdot 78 = \underline{46{,}8\,MPa}$$

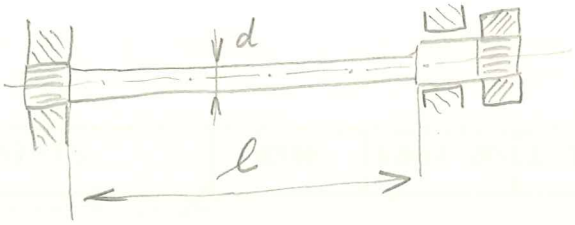
4.99 | $\tau_{Dk} = ?$ pro litinu 422432 přímka se vroubkovým ohybem

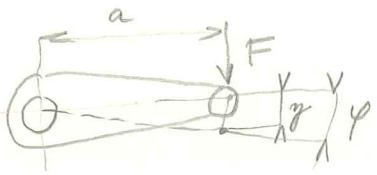
$\sigma_{Pt} = 320\,MPa$ $\quad k = 4 \div 10$; $c_{II} = 0{,}75$

$$\sigma_{Dt} = c_{II}\frac{\sigma_{Pt}}{k} = 0{,}75\,\frac{320}{4} = \underline{60\,MPa}$$

$\underline{\tau_{Dk} \doteq \sigma_{Dt}}$ pro litinu

4.100





$d = 16\,mm$

ocel 14 160.7

$k = 1{,}5$ míjivé zatížení

$\tau = ?$

$M_k = 200\,Nm$

$d = 16\,mm$

$\sigma_{Ck} = 883\,MPa$

$\sigma_{Pk} = 1128 \div 1324\,MPa$

$c_{II} = 0{,}7$

$$\sigma_{D t_{II}} = \frac{\sigma_{Ck}}{k} c_{II} = \frac{883}{1{,}5} 0{,}7 = 412\,MPa$$

$$\tau_{DK} = 0{,}6\,\sigma_{D t_{II}} = 0{,}6 \cdot 412 = \underline{247{,}2\,MPa}$$

$$\tau_{DK} \geq \tau_K = \frac{M_K}{W_K} = \frac{M_K}{\frac{\pi}{16} d^3} = \frac{16 \cdot 200 \cdot 10^3}{\pi \cdot 16^3} = \underline{248{,}6\,MPa}$$

$\tau_{DK} \doteq \tau_K$ pro $\tau_{DK} = 0{,}6 \sigma_{D t_{II}}$ bezpečně vyhovuje

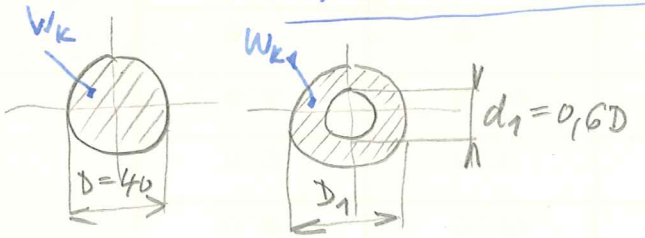
---

4.101 Plný válec $D = 40\,mm$ nahradíte dutým $d_1 = 0{,}6 D_1$

$D_1 = ?$ aby τ_{max} byl v obou průřezech stejné

Jaký bude poměr G_p a G_D (plného a dutého)

Řešte obecně.



$W_K = W_{K1}$

$$W_K = \frac{\pi}{16} D^3$$

$$W_{K1} = \frac{\pi}{16} \frac{D_1^4 (1 - 0{,}6^4)}{D_1} = \frac{\pi}{16} D_1^3 (1 - 0{,}6^4)$$

$$\frac{\pi}{16} D^3 = \frac{\pi}{16} D_1^3 (1 - 0{,}6^4)$$

$$D_1 = \sqrt[3]{\frac{D^3}{1 - 0{,}6^4}} = D \sqrt[3]{\frac{1}{1 - 0{,}6^4}} = 40 \sqrt[3]{\frac{1}{1 - 0{,}6^4}} = 40 \cdot 1{,}0473 = 41{,}89\,mm$$

$D_1 = 42\,mm$

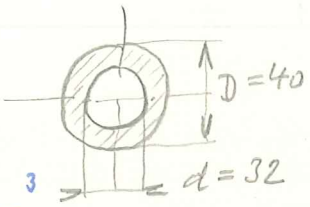
$$d_1 = 0{,}6 D_1 = 0{,}6 \cdot 42 = 25{,}13\,mm \doteq \underline{25{,}2\,mm}$$

$$G_p = \frac{\pi D^2}{4} l \varrho$$

$$G_D = \frac{\pi}{4} (D_1^2 - d_1^2) l \varrho$$

$$p = \frac{D^2}{D_1^2 - d_1^2} = \frac{40^2}{42^2 - 25{,}2^2} = \underline{1{,}472}$$

4.102 Trubka $D = 40\,mm$, $d = 32\,mm$ z oceli 11 370
přenáší výkon $P = 500\,W$; $\omega = 1\,s^{-1}$ – zatížení statické
$k = 2$, kontroly: $\tau_k = ?$



$$M_k = \frac{P}{\omega} = \frac{500}{1} = 500\,Nm$$

$$W_k = \frac{\pi}{16}\,\frac{D^4 - d^4}{D} = \frac{\pi}{16}\,\frac{40^4 - 32^4}{40} = 7419\,mm^3$$

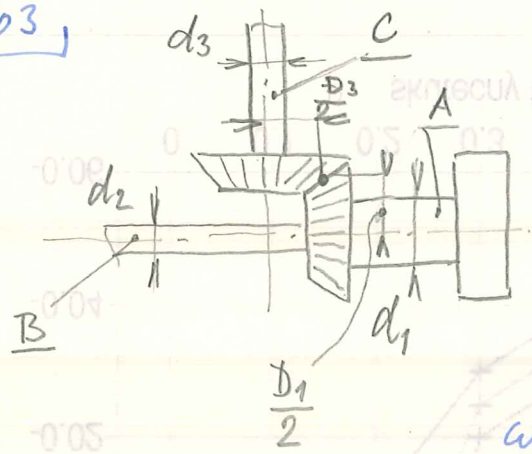
$$\tau_k = \frac{M_k}{W_k} = \frac{500 \cdot 10^3}{7419} = \underline{67{,}39\,MPa}$$

$$\tau_{Dk} = (0{,}6 \div 0{,}65)\,\sigma_{Dt} \qquad \sigma_{Dt} = \frac{0{,}6\,\sigma_{Pt}}{k} = \frac{0{,}6 \cdot 370}{2} = 111\,MPa$$

$$\tau_{Dk} = 0{,}65 \cdot 111 = \underline{72{,}15\,MPa}$$

$$\underline{\tau_k < \tau_{Dk} \text{ – vyhovuje}}$$

4.103



$P = 44\,kW$ $\quad D_1 = 600\,mm$
$n = 120\,min^{-1}$ $\quad D_2 = 240\,mm$
$\tau_{Dk} = 20\,MPa$
Ložiska C .... ½ P
B ... ½ P
$d_1 = ?,\ d_2 = ?,\ d_3 = ?$

$$\omega_A = 2\pi n = \frac{\pi n}{30} = \frac{\pi \cdot 120}{30} = 12{,}56\,s^{-1}$$

$$M_{kA} = \frac{P}{\omega_A} = \frac{44 \cdot 10^3}{12{,}56} = \underline{3503\,Nm}$$

$$M_{kB} = \frac{\frac{P}{2}}{\omega_B} = \frac{M_{kA}}{2} = \frac{3503}{2} = \underline{1751{,}5\,Nm} \qquad \omega_B = \omega_A$$

$$v_A = v_C$$

$$\omega_A r_1 = \omega_C r_3 \Longrightarrow \omega_C = \omega_A \frac{r_1}{r_3} = 12{,}56\,\frac{300}{120} = 31{,}4\,s^{-1}$$

$$M_{kC} = \frac{\frac{P}{2}}{\omega_C} = \frac{22 \cdot 10^3}{31{,}4} = \underline{700{,}6\,Nm}$$

$$\tau_{Dk} = \frac{M_{kA}}{W_{kA}} \Longrightarrow W_{kA} = \frac{M_{kA}}{\tau_{Dk}} = \frac{3503 \cdot 10^3}{20} = \underline{175{,}2 \cdot 10^3\,mm^3}$$

$$W_{kB} = \frac{M_{kB}}{\tau_{Dk}} = \frac{1751{,}5 \cdot 10^3}{20} = \underline{87{,}57 \cdot 10^3\,mm^3}$$

$$W_{kC} = \frac{M_{kC}}{\tau_{Dk}} = \frac{700{,}6 \cdot 10^3}{20} = \underline{35 \cdot 10^3\,mm^3}$$

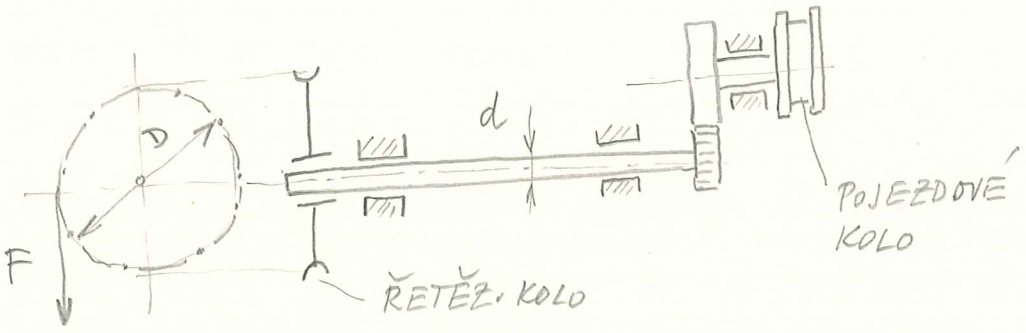
4.103) Pokračování

$$W_{kA} = \frac{\pi}{16} d_A^3 \Rightarrow d_A = \sqrt[3]{\frac{16 W_{kA}}{\pi}} = \sqrt[3]{\frac{16 \cdot 175{,}2 \cdot 10^3}{\pi}} = \underline{96{,}3 \text{ mm}}$$

$$d_B = \sqrt[3]{\frac{16 W_{kB}}{\pi}} = \sqrt[3]{\frac{16 \cdot 87{,}57 \cdot 10^3}{\pi}} = \underline{76{,}4 \text{ mm}}$$

$$d_C = \sqrt[3]{\frac{16 W_{kC}}{\pi}} = \sqrt[3]{\frac{16 \cdot 35 \cdot 10^3}{\pi}} = \underline{56{,}3 \text{ mm}}$$

4.104)



$F = 200\,N$ $\quad k = 2$ $\quad d = ?$

$D = 800\,mm$ $\quad c_{III} = 0{,}65$ $\quad$ mat. hřídele 11420

Rázy ... 65% σ_{Dt}

$$\sigma_{Dt_{III}} = c_{III} \frac{0{,}6\,\sigma_{Pt}}{k} = 0{,}65 \frac{0{,}6 \cdot 420}{2} = 81{,}9 \text{ MPa}$$

$$\sigma_{Dt_{III\,R}} = 0{,}65\,\sigma_{Dt_{III}} = 0{,}65 \cdot 81{,}9 = 53{,}2 \text{ MPa}$$

$$\tau_{Dk} = 0{,}6\,\sigma_{Dt_{III\,R}} = 0{,}6 \cdot 53{,}2 \doteq \underline{32 \text{ MPa}}$$

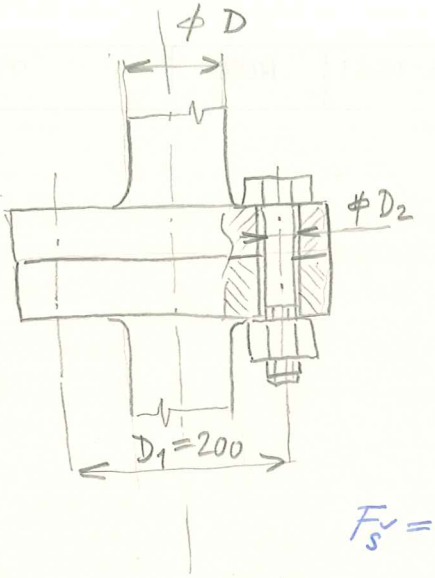
$$M_k = F \frac{D}{2} = 200 \frac{800}{2} = 80000 \text{ N mm}$$

$$W_k = \frac{M_k}{\tau_{Dk}} = \frac{80000}{32} = 2500 \text{ mm}^3$$

$$d = \sqrt[3]{\frac{16 W_k}{\pi}} = \sqrt[3]{\frac{16 \cdot 2500}{\pi}} = 23{,}25 \text{ mm}$$

$$\underline{d = 25 \text{ mm}}$$

4.105 |



$D = 100\ mm$ $\quad i = ?$ počet šroubů
$D_1 = 200\ mm$
$D_2 = 20\ mm$
$\tau_{Ds} = 60\ MPa$
$\tau_{Dk} = 70\ MPa$

únosnost šroubu:

$$M_k = \tau_{Dk} \cdot W_k = \tau_{Dk} \frac{\pi D^3}{16} = 70 \frac{\pi \cdot 100^3}{16} = 13,74 \cdot 10^6 \ N \cdot mm$$

$$F_s = \frac{M_k}{\frac{D_1}{2}} = \frac{2 \cdot 13,74 \cdot 10^6}{200} = 13,74 \cdot 10^4 \ N$$

$$\tau_{Ds} = \frac{F}{i\,S} \Rightarrow i = \frac{F}{\tau_{Ds} S} = \frac{F}{\tau_{Ds} \frac{\pi d^2}{4}} = \frac{4F}{\tau_{Ds} \pi D_2^2} = \frac{4 \cdot 13,74 \cdot 10^4}{60\,\pi \cdot 20^2}$$

$$= 7,29$$

$i = 8$ šroubů

4.106 | Hřídel $D = 20\ mm$ zatížen $M_k = 120\ Nm$ $\quad \tilde{\varphi} = ?$ $\quad \varphi^\circ = ?$
$l = 0,9\ m$
$E = 2,06 \cdot 10^{11}\ Pa$
$m = 3,45$

$$G = \frac{E}{2(1+\mu)} = \frac{E}{2\left(1+\frac{1}{m}\right)} = \frac{2,06 \cdot 10^5}{2\left(1+\frac{1}{3,45}\right)} = 0,795 \cdot 10^5\ MPa$$

$$\tilde{\varphi} = \frac{M_k l}{G J_p} = \frac{120 \cdot 10^3 \cdot 900}{0,795 \cdot 10^5 \cdot 15\,700} = 8,61 \cdot 10^{-2}\ rad$$

$$J_p = \frac{\pi}{32} D^4 = \frac{\pi}{32} \cdot 20^4 = 15\,700\ mm^4$$

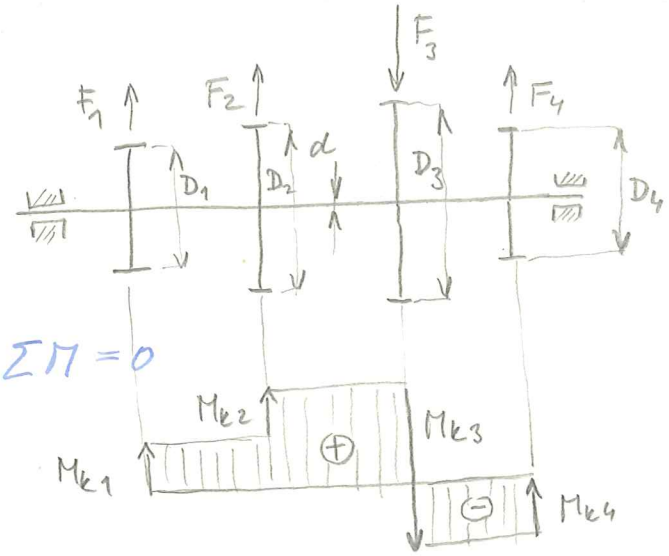
$$\varphi^\circ = \frac{180}{\pi} \tilde{\varphi} = \frac{180}{\pi} 8,61 \cdot 10^{-2} = 4,93^\circ$$

$$\varphi^\circ = \varphi^\circ \frac{1000}{900} = 5,47^\circ$$

hřídel délky 1m bude mít úhel zkroucení $5,47^\circ$

4. 107 | 2.

$d = ?$ hřídele



$\tau_{DK} = 50\,MPa$ $\quad D_1 = 0{,}6\,m$

$\vartheta^\circ = 0{,}33^\circ \cdot m^{-1}$ $\quad D_2 = 0{,}8\,m$

$G = 0{,}8 \cdot 10^{11}\,Pa$ $\quad D_4 = 1{,}2\,m$

$D_3 = 1\,m$

$F_3 = 2500\,N$ – obvod. síla

$F_1 = F_2 = F_4 = F$ – obvod. síly

$\Sigma M = 0$

$$F_3 \frac{D_3}{2} = F_1 \frac{D_1}{2} + F_2 \frac{D_2}{2} + F_4 \frac{D_4}{2}$$

$$F_3 \frac{D_3}{2} = F\left(\frac{D_1}{2} + \frac{D_2}{2} + \frac{D_4}{2}\right) \Rightarrow F = F_3 \frac{D_3}{D_1 + D_2 + D_3} =$$

$$= 2500 \frac{1}{0{,}6 + 0{,}8 + 1{,}2} = 961{,}5\,N$$

$$M_{k1} = F \frac{D_1}{2} = 961 \frac{0{,}6}{2} = 288{,}45\,Nm$$

$$M_{k2} = F \frac{D_2}{2} = 961 \frac{0{,}8}{2} = 384{,}6\,Nm$$

$$M_{k3} = F_3 \frac{D_3}{2} = 2500 \frac{1}{2} = 1250\,Nm$$

$$M_{k4} = F \frac{D_4}{2} = 961 \frac{1{,}2}{2} = 576{,}3\,Nm$$

$$M_{k\,max} = M_{k1} + M_{k2} = 288{,}45 + 384{,}6 = 673\,Nm$$

$$\vartheta^\circ = \frac{180}{\pi} \frac{M_{k\,max}\, l}{G J_p} \Rightarrow J_p = \frac{180}{\pi} \frac{M_{k\,max} \cdot l}{G \vartheta}$$

ϑ .... 1 m

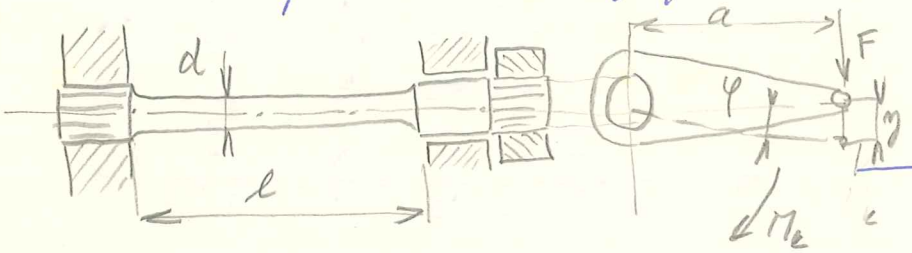
$$J_p = \frac{180}{\pi} \frac{673 \cdot 10^3 \cdot 1000}{0{,}8\,10^5 \cdot 0{,}33} = 1460608\,mm^4$$

$$J_p = \frac{\pi}{32} d^4 \rightarrow d = \sqrt[4]{\frac{32 J_p}{\pi}} = \sqrt[4]{\frac{32 \cdot 1460608}{\pi}} = 62{,}1\,mm$$

2

4.108) $\widehat{\varphi} = ?$ $y = ?$ $\varphi^\circ = ?$ údaje jsou z 4.100

$l = 1\,m$
$G = 0{,}8 \cdot 10^5\ MPa$
$d = 16\,mm$
$M_k = 200 \cdot 10^3\ Nmm$



$$J_p = \frac{\pi}{32} \cdot d^4 = \frac{\pi}{32} \cdot 16^4 = 64\,339\ mm^4$$

$\widehat{\varphi} = \sqrt{\varphi}$ pro $l = 1m$
$\varphi^\circ = \sqrt{\varphi}^\circ$

$$\widehat{\varphi} = \frac{M_k \cdot l}{G J_p} = \frac{200 \cdot 10^3 \cdot 1000}{0{,}8 \cdot 10^5 \cdot 64\,339} = \underline{0{,}3885\ rad}$$

$$\varphi^\circ = \frac{180}{\pi} \widehat{\varphi} = \frac{180}{\pi} \cdot 0{,}3885 = \underline{22{,}25^\circ}$$

$$y = a \cdot \widehat{\varphi} = 200 \cdot 0{,}3885 = \underline{77{,}7\ mm}$$

4.109) Kolikrát je větší úhel zkroucení z př. 4.108, když

$M_k = 200 \cdot 10^3\ Nmm$; $G = 0{,}8 \cdot 10^5\ MPa$
$l = 1\,m$

$d_1 = 2d = 32\,mm$

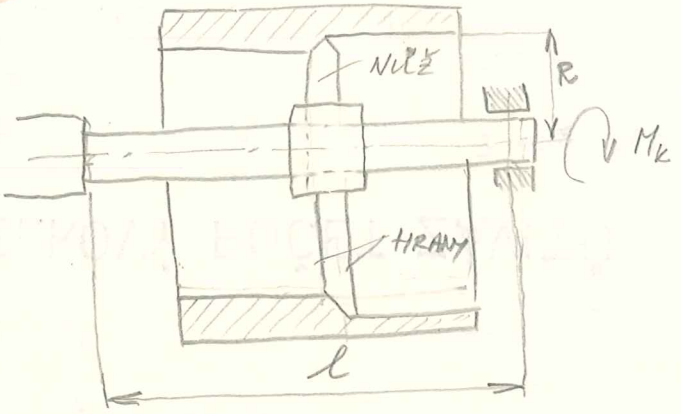
$$J_p = \frac{\pi}{32} d_1^4 = \frac{\pi}{32} 32^4 = 102\,943\ mm^4$$

$$\widehat{\varphi}_1 = \frac{M_k l}{G J_p} = \frac{200 \cdot 10^3 \cdot 1000}{0{,}8 \cdot 10^5 \cdot 102\,943} = 0{,}0243\ rad$$

$$\varphi_1^\circ = \frac{180}{\pi} \cdot \widehat{\varphi} = \frac{180}{\pi} 0{,}0243 = 1{,}39^\circ$$

$$\frac{\varphi^\circ}{\varphi_1^\circ} = \frac{22{,}25}{1{,}39} = 16{,}18 \qquad \underline{\varphi^\circ \doteq 16\,\varphi_1^\circ}$$

4.110 Vyvrtávací tyč $d = ?$



$F = 10^4\ N$
$R = 200\ mm$
$l = 1{,}2\ m$
$G = 0{,}77 \cdot 10^5\ MPa$
$\varphi^\circ = 0{,}5$

$M_k = F \cdot R$

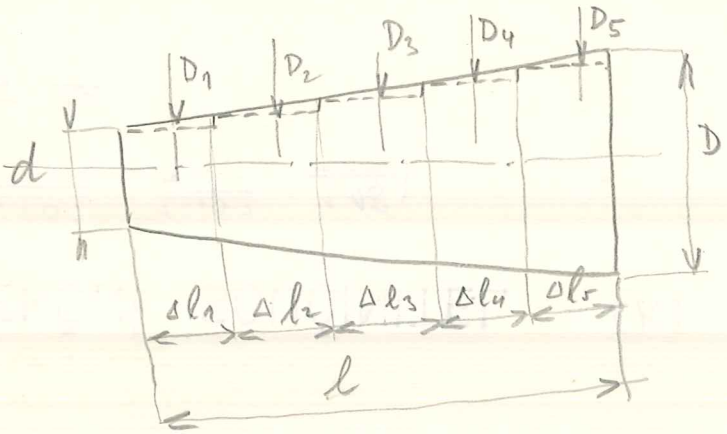
$$\varphi^\circ = \frac{M_k l}{G J_p} \frac{180}{\pi}$$

$$J_p = \frac{M_k l}{G \varphi} \frac{180}{\pi} = \frac{F R l}{G \varphi} \frac{180}{\pi} =$$

$$= \frac{10^4 \cdot 200 \cdot 1200}{0{,}77 \cdot 10^5 \cdot 0{,}5} \frac{180}{\pi} = 35\,71\,790 = \underline{\underline{357{,}18 \cdot 10^4\ mm^4}}$$

$$J_p = \frac{\pi}{32} d^4 \quad \dots \quad \underline{\underline{d = \sqrt[4]{\frac{32 J_p}{\pi}} = \sqrt[4]{\frac{32 \cdot 357{,}18 \cdot 10^4}{\pi}} = 77{,}66\ mm}}$$

4.111



$\bar{\varphi} = ?$
$l = 0{,}5\ m$
$M_k = 100\ Nm$
$d = 20\ mm$
$G = 0{,}8 \cdot 10^5\ MPa$

$$\Delta l = \frac{l}{5}$$

$$\varphi_1 = \frac{M_k \Delta l_1}{G J_{p1}} \quad \dots\dots$$

$$\bar{\varphi}_{celk} = \frac{M_k \Delta l}{G} \left(\frac{1}{J_{p1}} + \frac{1}{J_{p2}} + \frac{1}{J_{p3}} + \frac{1}{J_{p4}} \right) = \frac{M_k \Delta l\, 32}{G \pi} \left(\frac{1}{d_1^4} + \dots \frac{1}{d_5^4} \right)$$

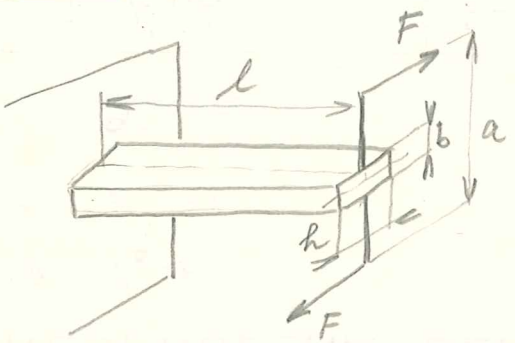
$d_1 = 20$	$\frac{1}{d_1^4} = 6{,}25 \cdot 10^{-6}$
$d_2 = 25$	$\frac{1}{d_2^4} = 2{,}56 \cdot 10^{-6}$
$d_3 = 30$	$\frac{1}{d_3^4} = 1{,}2345 \cdot 10^{-6}$
$d_4 = 35$	$\frac{1}{d_4^4} = 6{,}6638 \cdot 10^{-7}$
$d_5 = 40$	$\frac{1}{d_5^4} = 3{,}906 \cdot 10^{-7}$

$$\Sigma = 1{,}110148 \cdot 10^{-5}\ mm^{-4}$$

$$\bar{\varphi} = \frac{100 \cdot 10^3 \cdot 100 \cdot 32}{0{,}77 \cdot 10^5 \cdot \pi} \cdot 1{,}110148 \cdot 10^{-5} = \underline{\underline{0{,}01469\ rad}}$$

$$\varphi^\circ = \frac{180}{\pi} \bar{\varphi} = \frac{180}{\pi} 0{,}01469 = \underline{\underline{0{,}842^\circ}}$$

4.112



$l = 500\,mm$
$b = 32\,mm$
$h = 48\,mm$
$F = 2700\,N$
$a = 192\,mm$
$\tau_{Dk} = 46\,MPa$
$G = 0{,}8 \cdot 10^5\,MPa$

$\vartheta_k = ?$
$\varphi = ?$
$\vartheta = ?$

$M_k = F \cdot a = 2700 \cdot 192 = 518\,400\,Nmm$

$\tau_{k\,max} = \frac{M_k}{\alpha b^2 h} = \frac{518\,400}{0{,}231 \cdot 32^2 \cdot 48} = 45{,}65\,MPa < \tau_{Dk}$

$\varphi = \frac{M_k l}{\gamma G b^3 h} = \frac{518\,400 \cdot 500}{0{,}196 \cdot 0{,}8 \cdot 10^5 \cdot 32^3 \cdot 48} = 0{,}0112\,rad$

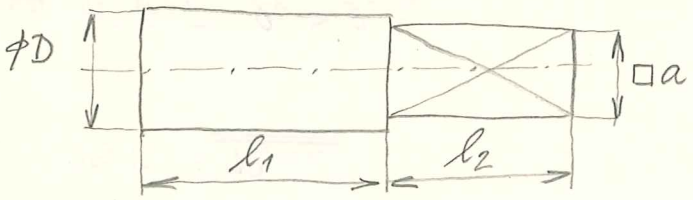
$\vartheta = \frac{\varphi}{l} = \frac{0{,}0112}{0{,}5} = 0{,}02242\,rad$

$\frac{h}{b} = \frac{48}{32} = 1{,}5$

$\alpha = 0{,}231$

$\gamma = 0{,}196$

4.113



$l_1 = 600\,mm$
$D = 20\,mm$
$l_2 = ?$
$\square a = ?$
$M_k = 80\,Nm$
$G = 0{,}8 \cdot 10^{11}\,Pa$

a) strana $\square a = ?$
b) $l_2 = ?$ když $\varphi_\phi = \varphi_\square$

$\tau_k = \frac{M_k}{W_k} = \frac{16 M_k}{\pi D^3} = \frac{16 \cdot 80 \cdot 10^3}{\pi \cdot 20^3} = 50{,}92\,MPa$

$\frac{l}{b} = 1$ → $\alpha = 0{,}208$, $\gamma = 0{,}141$

a) $\tau_k = \frac{M_k}{\alpha a^3} \Rightarrow a = \sqrt[3]{\frac{M_k}{\alpha \tau_k}} = \sqrt[3]{\frac{80 \cdot 10^3}{0{,}208 \cdot 50{,}92^3}} = 19{,}62\,mm$

b) $\varphi_\phi = \frac{M_k l_1}{G J_p} = \frac{80 \cdot 10^3 \cdot 600}{0{,}8 \cdot 10^5 \cdot 15\,707{,}9} = 0{,}03819\,rad$

$J_p = \frac{\pi}{32} d^4 = \frac{\pi}{32} \cdot 20^4 = 15\,707{,}9\,mm^4$

$\varphi_\phi = \varphi_{\square a} = \frac{M_k l_2}{\gamma G a^4}$

$l_2 = \frac{\varphi G \gamma a^4}{M_k} = \frac{0{,}03819 \cdot 0{,}8 \cdot 10^5 \cdot 0{,}141 \cdot 19{,}62^4}{80 \cdot 10^3} = 797{,}92\,mm$

4.114

$b = ?$ $\quad \frac{b}{h} = 0{,}5$ $\quad P = 2000\,W$ $\quad \tau_{Dk} = 50\,MPa$

$h = ?$ $\quad n = 1\,s^{-1}$

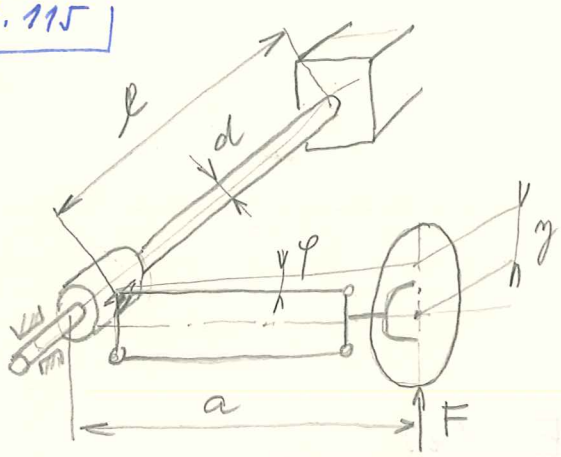
$$M_k = \frac{P}{\omega} = \frac{P}{2\pi n} = \frac{2000}{2\pi \cdot 1} = 318{,}3\,Nm = \underline{318{,}3 \cdot 10^3\,Nmm}$$

$$\tau_{Dk} = \frac{M_k}{\alpha b^2 h} \quad \dots \; b = 0{,}5h \; ; \; \frac{h}{b} = 2 \dots \alpha = 0{,}246$$

$$\tau_{Dk} = \frac{M_k}{\alpha\, 0{,}5^2 h^2 \cdot h} \longrightarrow \underline{h} = \sqrt[3]{\frac{M_k}{\alpha \cdot 0{,}5^2\, \tau_{Dk}}} = \sqrt[3]{\frac{318{,}3 \cdot 10^3}{0{,}246 \cdot 0{,}5^2 \cdot 50}} = \underline{\underline{46{,}95\,mm}}$$

$$b = 0{,}5h = 0{,}5 \cdot 46{,}95 = \underline{\underline{23{,}47\,mm}}$$

4.115



$a = 300\,mm$ $\quad y = ?$

$\varphi = 25^\circ$ $\quad d = ?$

$\tau_{Dk} = 500\,MPa$

$G = 0{,}83 \cdot 10^5\,MPa$

$F = 3000\,N$

Rázy ... M_k zvětšit o 50%

$$M_k = 1{,}5\,Fa = 1{,}5 \cdot 3000 \cdot 300 = 1350\,000\,Nmm$$

$$W_k = \frac{M_k}{\tau_{Dk}} = \frac{1350\,000}{500} = 2700\,mm^3$$

$$W_k = \frac{\pi}{16} d^3$$

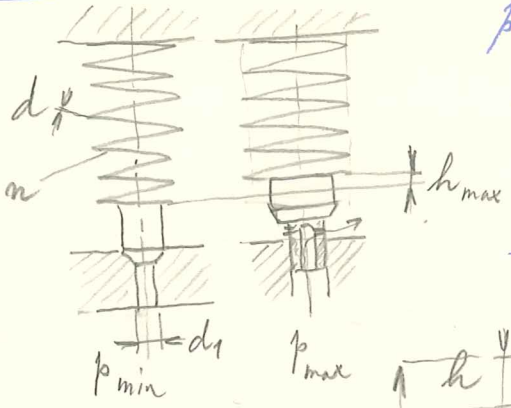
$$d = \sqrt[3]{\frac{16 W_k}{\pi}} = \sqrt[3]{\frac{16 \cdot 2700}{\pi}} = 23{,}95\,mm$$

$$\underline{\underline{d = 24\,mm}}$$

$$\varphi^\circ = \frac{180}{\pi} \frac{M_k l}{G J_p} \qquad J_p = \frac{\pi}{32} d^4 = \frac{\pi}{32} \cdot 24^4 = 32\,572\,mm^4$$

$$l = \frac{\varphi^\circ \pi G J_p}{180\, M_k} = \frac{25 \cdot \pi \cdot 0{,}83 \cdot 10^5 \cdot 32\,572}{180 \cdot 1350\,000} = \underline{\underline{873{,}78\,mm}}$$

4. 116



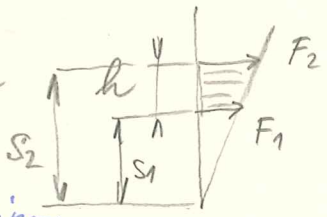
$p_{min} = 75.10^4\ Pa$ $\quad \tau_{Dk} = 400\ MPa$

$p_{max} = 90.10^4\ Pa$ $\quad G = 0{,}8.10^5\ MPa$

$h = 9\ mm$ $\quad d = ?$

$D_s = 24\ mm$ $\quad n = ?$

$d_1 = 16\ mm$



$$F_1 = \frac{\pi d_1^2}{4} \cdot p_{min} = \frac{\pi . 16^2}{4} \cdot 75.10^4 . 10^{-6} = 150{,}79\ N$$

$$F_2 = \frac{\pi d_1^2}{4} p_{max} = \frac{\pi . 16^2}{4} \cdot 90.10^{-2} = 180{,}95\ N$$

Tuhost prasiny

$$k = \frac{F_2 - F_1}{h} = \frac{180{,}95 - 150{,}79}{9} = 3{,}351\ N.mm^{-1}$$

Odhadem $i = 8$ $\quad$ ČSN 221

Korekce na ohyb $K = \frac{i + 0{,}2}{i - 1} = \frac{8{,}2}{7} = 1{,}71$

$$d = 2\sqrt[3]{\frac{F_1 D_s K}{\pi \tau_{Dk}}} = 2\sqrt[3]{\frac{150{,}79 . 24 . 1{,}71}{\pi . 400}} = 2 . 1{,}7 = 3{,}4\ mm$$

$d = 3{,}55\ mm$ ... δ_{SN}

$$S_1 = \frac{F_1}{k} = \frac{150{,}79}{3{,}351} = 44{,}99\ mm$$

$$S_2 = S_1 + h = 44{,}99 + 9 = 53{,}99\ mm$$

$$n = \frac{S_2 G d^4}{8 F_2 D_s^2} = \frac{53{,}99 . 0{,}8 . 10^5 . 3{,}55^4}{8 . 180{,}95 . 24^2} = 34{,}27 \text{ závitů činných}$$

4.117 Torsní obdélníková prašina

$l = 300\,mm$
$\tau_{Dk} = 400\,MPa$
$G = 0{,}83 \cdot 10^5\,MPa$
$M_k = 150\,Nm$
$a = ?$
$\varphi = ?$

$$W_k = \frac{M_k}{\tau_{Dk}} = \frac{150 \cdot 10^3}{400} = 375\,mm^3$$

$$W_k = \alpha a^3$$

$\frac{h}{b} = 1 \ldots\ldots \alpha = 0{,}208$
$\beta = 0{,}141$

$$a = \sqrt[3]{\frac{W_k}{\alpha}} = \sqrt[3]{\frac{375}{0{,}208}} = 12{,}17\,mm \qquad a = 12\,mm$$

$$\bar{\varphi} = \frac{M_k l}{\beta G a^4} = \frac{150 \cdot 10^3 \cdot 300}{0{,}141 \cdot 0{,}83 \cdot 10^5 \cdot 12^4} = 0{,}1854\,rad$$

$$\varphi^\circ = \frac{180}{\pi}\bar{\varphi} = \frac{180}{\pi} \cdot 0{,}1854 = 10{,}62^\circ$$

4.118 Není zadáno h

$$k = \frac{F_{max} - F_{min}}{h} = \frac{120 - 90}{2} = \frac{30}{2} = 15 \ N \cdot mm^{-1}$$

Odhaden $i = 10$

$$K = \frac{i + 0{,}2}{i - 1} = \frac{10{,}2}{9} = 1{,}13$$

$$d = 2 \sqrt[3]{\frac{F_{min} \cdot D_s \cdot K}{\pi \cdot \tau_k}} = 2 \sqrt[3]{\frac{90 \cdot 40 \cdot 1{,}13}{\pi \cdot 440}} = 2 \cdot 1{,}43 = 2{,}86 \Rightarrow$$

$\Phi 3$ mm

$$S_1 = \frac{F_{min}}{k} = \frac{90}{15} = 6 \ mm$$

$$S_2 = S_1 + h = 6 + 2 = 8 \ mm$$

$$n = \frac{S_2 \cdot G \cdot d^4}{8 \cdot F_{max} \cdot D_s^2} = \frac{8 \cdot 85 \cdot 10^5 \cdot 3^4}{8 \cdot 120 \cdot 40^2} = 35{,}86 \ \check{c}. \ z\acute{a}v.$$

118

$h = 2$ mm

4.119

$l = 6\,m \qquad \alpha = 60°$

$M_k = 1500\,Nm$

$\varphi = \frac{\alpha}{100} = \frac{60}{100} = 0{,}6°$

$G = 0{,}8 \cdot 10^5\,MPa$

$\tau_{DK} = 30\,MPa$

$d = ?$

$$\varphi° = \frac{180}{\pi}\frac{M_k l}{G J_p} \Rightarrow J_p = \frac{180}{\pi}\frac{M_k l}{G \varphi°} = \frac{180}{\pi}\frac{1500 \cdot 10^3 \cdot 6000}{0{,}8 \cdot 10^5 \cdot 0{,}6}$$

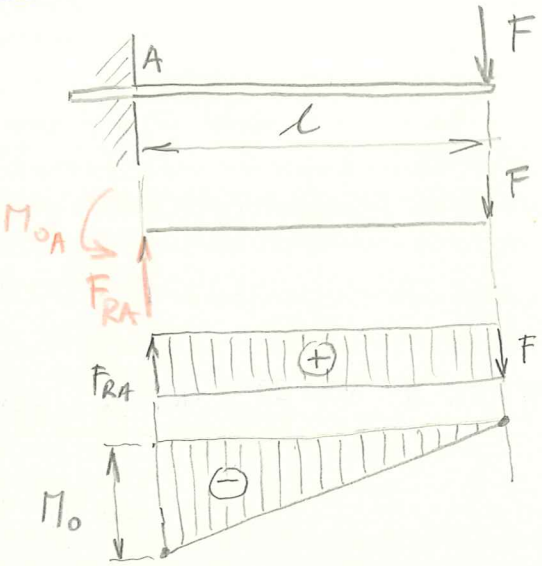
$$= 10742958\,mm^4$$

$$d = \sqrt[4]{\frac{32 J_p}{\pi}} = \sqrt[4]{\frac{32 \cdot 10742958}{\pi}} = \underline{102{,}27\,mm}$$

$$\tau_k = \frac{M_k}{W_k} = \frac{16 M_k}{\pi d^3} = \frac{16 \cdot 10742958}{\pi \cdot 102{,}27^3} = \underline{7{,}14\,MPa}$$

$$\underline{\tau_k < \tau_{DK}}$$

4.120 Mrchnete nosník – zobrazte posouv. síla a ohyb. moment



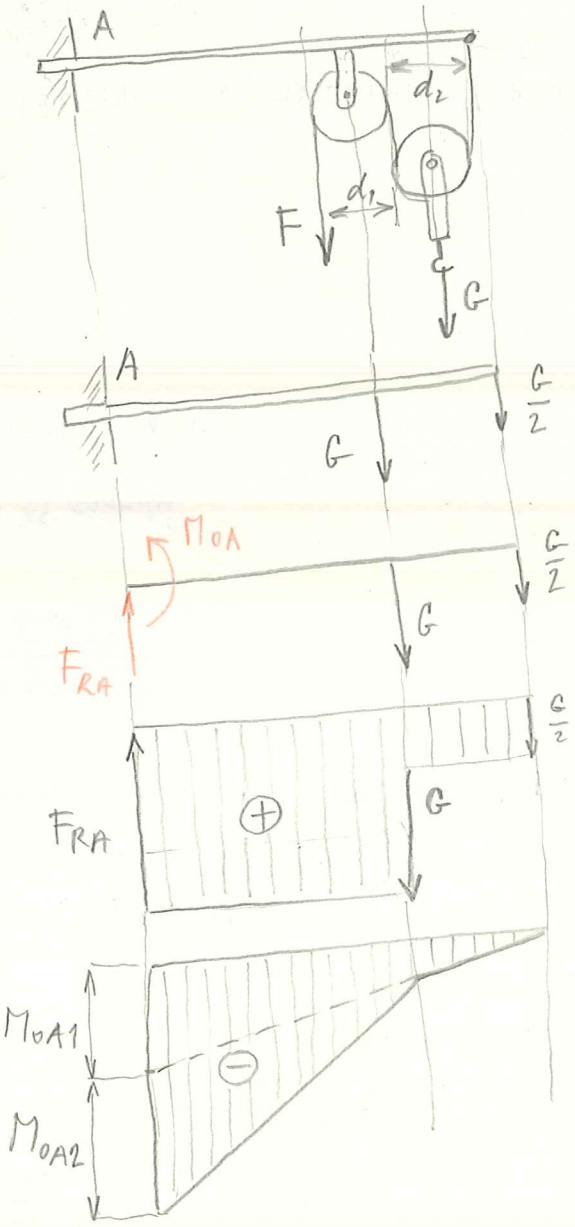
$F = 1000\,N$

$l = 1\,m$

$F_{RA} = 1000\,N$

$M_o = F.l = 1000.1 = \underline{1000\,Nm}$

4.121



$d_1 = d_2 = 160\,mm$

$l = 1\,m$

$G = 400\,N$

Mrchnete nosník, určete zatížení a nakreslete obr. posouv. sil a momentů.

$F_{RA} = G + \frac{G}{2} = \frac{3}{2}G = \frac{3}{2}\cdot 400 = \underline{600\,N}$

$M_{oA1} = \frac{G}{2}.l = \frac{400}{2}.1000 = \underline{20.10^3\,Nmm}$

$M_{oA2} = G\left(l - d_2 - \frac{d_1}{2}\right)$

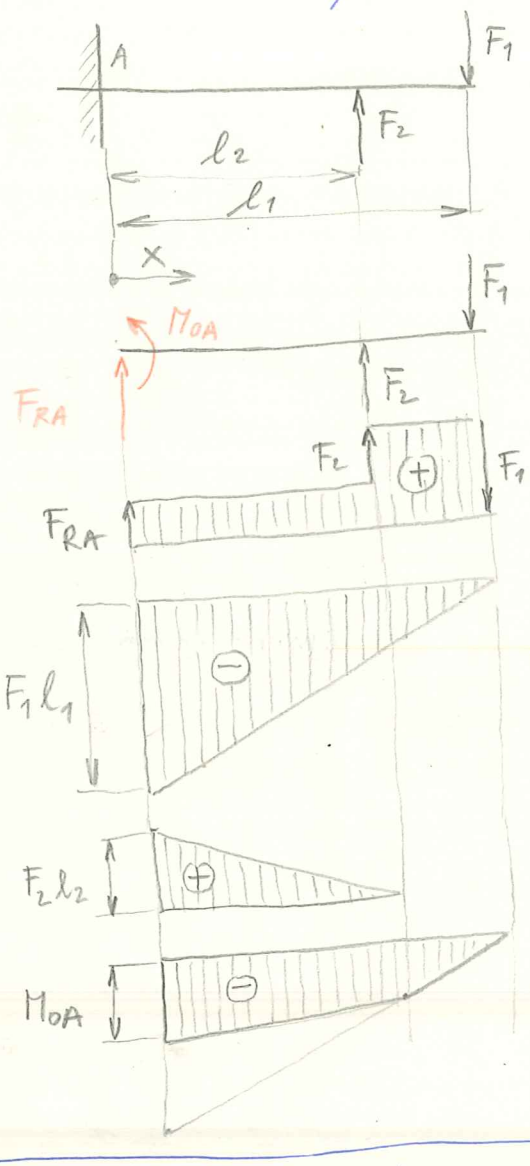
$= G\,(l - 240) = 400\,(1000 - 240)$

$= \underline{304.10^3\,Nmm}$

$M_{oA} = M_{oA1} + M_{oA2} = 10^3\,(304 + 20)$

$= \underline{324.10^3\,Nmm}$

4.122 Vyřešte nosník, určete reakce, vnitřní síly a ohyb. moment, nakreslete obrazce prům. sil a momentů



$F_1 = 1000\,N$

$F_2 = 600\,N$

$l_1 = 800\,mm$

$l_2 = 500\,mm$

$\Sigma F_y = 0 \dots F_{RA} + F_2 - F_1 = 0$

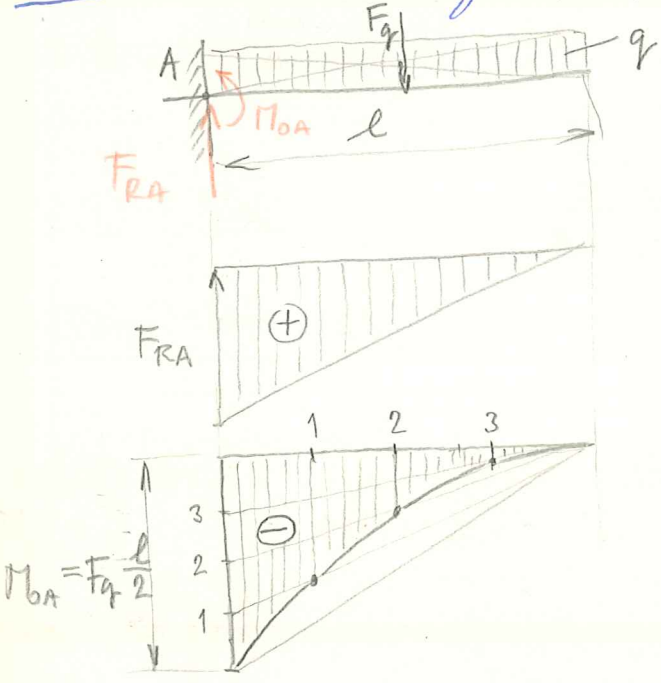
$F_{RA} = F_1 - F_2 = 1000 - 600 = \underline{400\,N}$

$\Sigma M_A = 0$

$M_{oA} = F_1 l - F_2 l_2 =$

$= 1000 \cdot 800 - 600 \cdot 500 = \underline{500 \cdot 10^3\,Nmm}$

4.123 Vetknutý nosník je zatížen spojitým zatížením $q = 400\,N.m^{-1}$ v celé délce $l = 3\,m$. Analyticky určete průběh sil a momentů



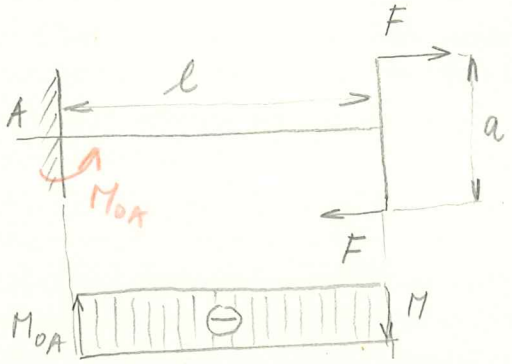
$F_q = q \cdot l = 400 \cdot 3 = \underline{1200\,N}$

$\Sigma F_y = 0 \dots F_{RA} = F_q$

$M_{oA} = F_q \cdot \frac{l}{2} = q \frac{l^2}{2} = 400 \frac{3^2}{2} =$

$= 1800\,Nm = \underline{1800 \cdot 10^3\,Nmm}$

4.124



$F = 2000\,N$ Posuv. síla, ohyb. moment – určete

$a = 0{,}4\,m$

$l = 1\,m$

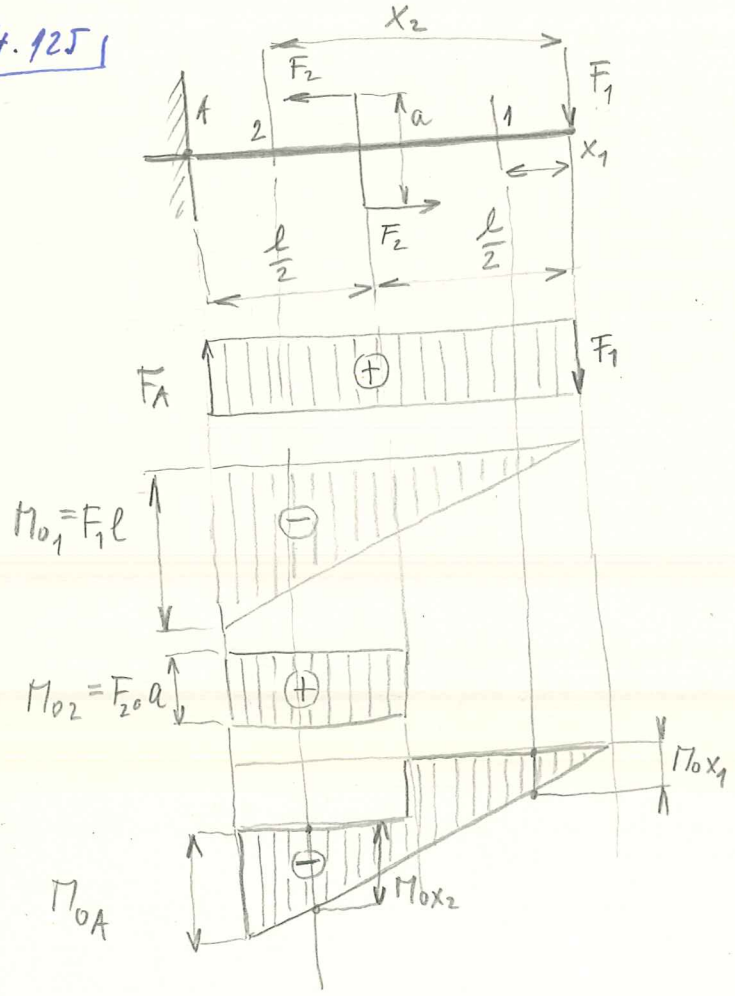
$\Sigma F_x = 0 \ldots F - F = 0$

$\Sigma F_y = 0 \ldots$ není žádná síla

$\Sigma M_A = 0 \ldots -F.a = M_{oA}$

$M_{oA} = -F.a = -2000.\,0{,}4.10^3 = \underline{\underline{-800.10^3\,Nm}}$

4.125



$F_1 = 1000\,N$ $X_1 = 0{,}25\,m$

$F_2 = 500\,N$ $X_2 = 0{,}6\,m$

$a = 0{,}6\,m$ $M_{o\,max} = ?$

$l = 1\,m$ $M_{o1} = ?$

$M_{o2} = ?$

$F_A = F_1 = \underline{1000\,N}$

$M_{o1} = F_1 l = 1000.10^3 = 1000.10^3\,Nmm$

$M_{o2} = F_2.a = 500.\,0{,}6.10^3 = 300.10^3\,Nmm$

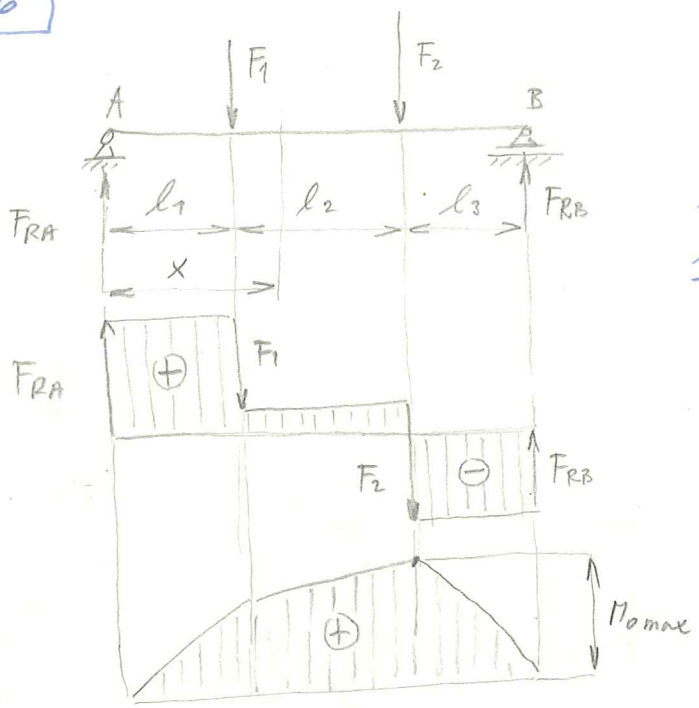
$M_{oA} = -M_{o1} + M_{o2} = (-1000+300).10^3$

$= \underline{\underline{-700.10^3\,Nmm}}$

$M_{oX_1} = F_1 X_1 = -1000.\,0{,}25.10^3 = \underline{-250.10^3\,Nmm}$

$M_{oX_2} = (-1000.\,0{,}6 + 300).10^3 = \underline{\underline{-300\,Nm}}$

4. 126)



$F_1 = 28\,000\ N$
$F_2 = 40\,000\ N$
$l_1 = 2{,}3\ m$
$l_2 = 3\ m$
$l_3 = 2{,}7$
$x = 3{,}6\ m$

$F_{RA}, F_{RB} = ?$
obrazce F_t, M_o
$M_{omax} = ?$
$F_{tx} = ?$
$F_{tx_1} = 0$ místo $x_1 = ?$

$\Sigma F_y = 0 \dots F_{RA} - F_1 - F_2 + F_{RB} = 0$
$\Sigma M_A = 0 \dots -F_1 l_1 - F_2 (l_1 + l_2) + F_{RB}(l_1 + l_2 + l_3) =$

$$F_{RB} = \frac{F_1 l_1 + F_2 (l_1 + l_2)}{l_1 + l_2 + l_3} = \frac{28 \cdot 2{,}3 + 40(2{,}3 + 3)}{2{,}3 + 3 + 2{,}7}$$

$$= 34{,}55\ kN$$

$F_{RA} = F_1 + F_2 - F_{RB} = 28 + 40 - 34{,}55 =$

$$= 33{,}45\ kN$$

$M_{omax} = F_{RA}(l_1 + l_2) - F_1 l_2 =$

$= 33{,}45 \cdot 5{,}3 - 28 \cdot 3 = 93{,}285\ kNm$

$F_{tx} = F_{RA} - F_1 = 33{,}45 - 28 = 5{,}45\ kN$

$x_1 = l_1 + l_2 = 5{,}3\ m$

$M_{ox} = F_{RA} \cdot x - F_1 (x - l_1) = 33{,}45 - 28(3{,}6 - 2{,}3) = 84{,}02\ kNm$

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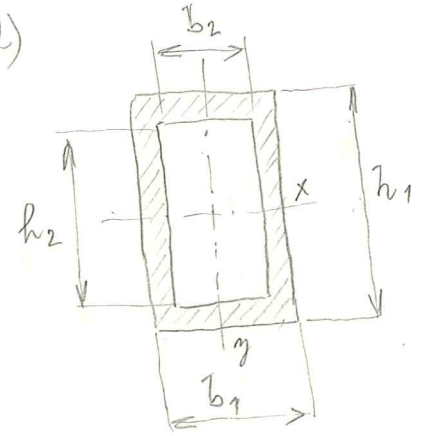
2)

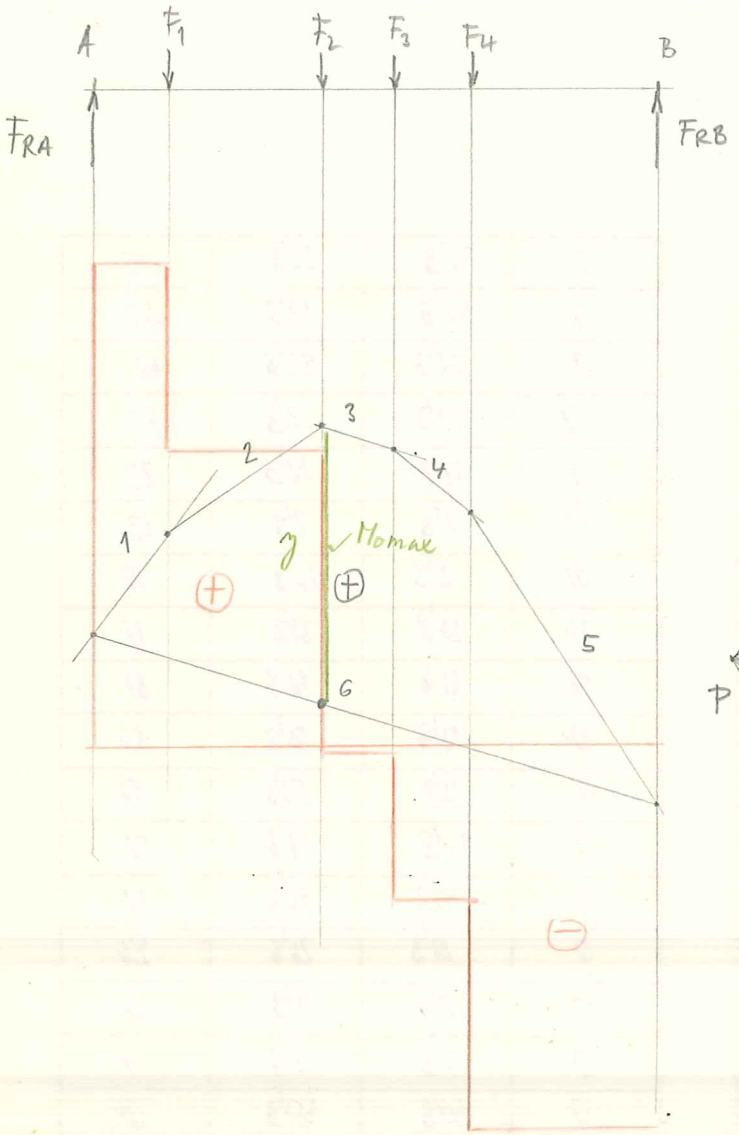


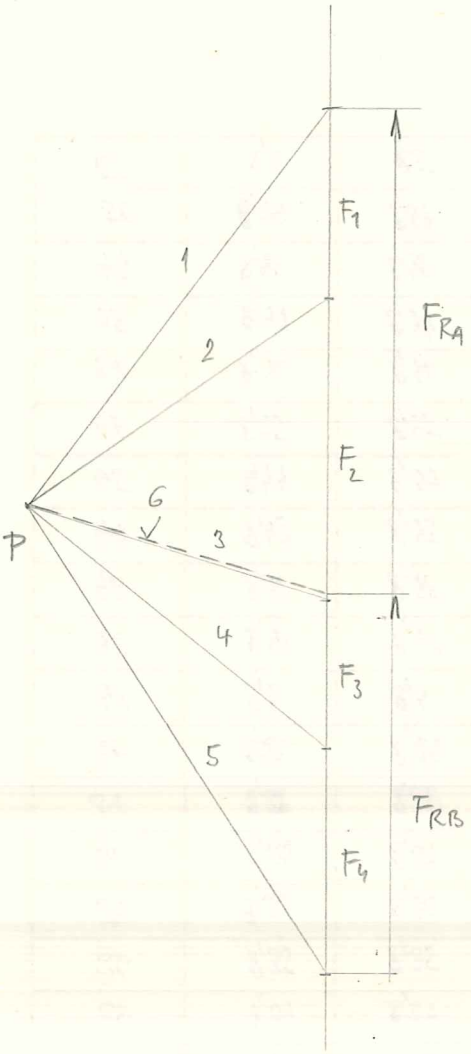
$h_1 = 60\ mm$
$h_2 = 50\ mm$
$b_1 = 40\ mm$
$b_2 = 30\ mm$

$W_{ox} = ?$

4.127 | Zadání

$H = 40\ mm$



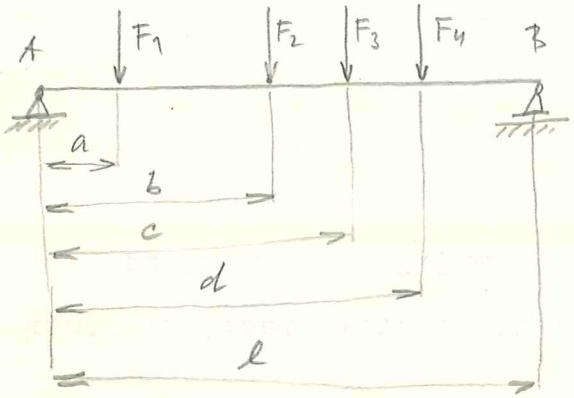


$y = 37\ mm$

$M_{omax} = y \cdot H \cdot m_L \cdot m_F = 37 \cdot 40 \cdot 100 \cdot 200 = 29600000\ Nmm$

$= 29600\ Nm$

Zadání:



$F_1 = 5000\ N$ $\quad a = 1\ m$

$F_2 = 8000\ N$ $\quad b = 3\ m$

$F_3 = 4000\ N$ $\quad c = 4\ m$

$F_4 = 6000\ N$ $\quad d = 5\ m$

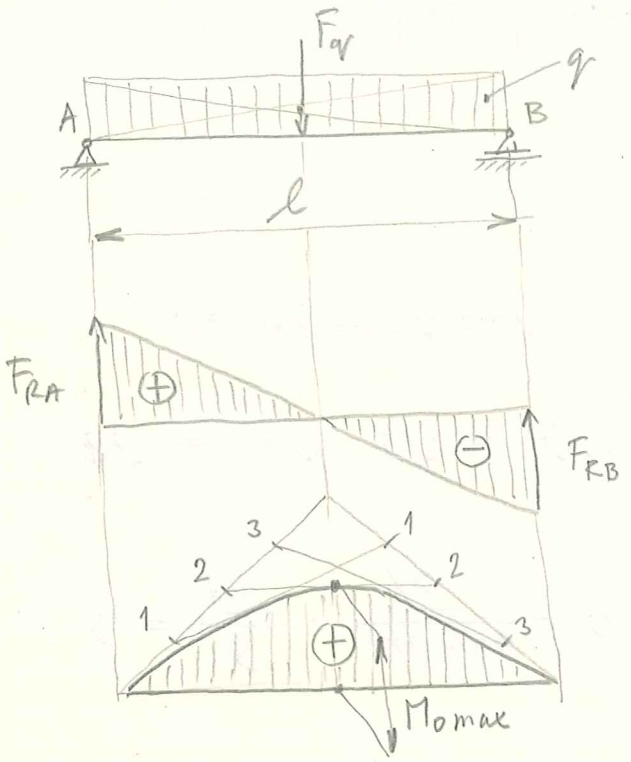
$l = 7{,}5\ m$

$m_L = 1:100 \ \ldots \ 1\ mm \triangleq 100\ mm$

$m_F = 1:200 \ \ldots \ 1\ mm \triangleq 200\ N$ ← Lépe 1:400

$H = 40\ mm$

4.128



$l = 1\,m$

$q = 400\,N\,m^{-1}$

Nakreslí průběh F_t, M_o

$M_{o\,max} = ?$

$\Sigma F_y = 0 \dots F_{RA} - F_q + F_{RB} = 0$ $\quad F_q = q \cdot l = 400$

$\Sigma M_A = 0 \dots -F_q \frac{l}{2} + F_{RB} \cdot l = 0$

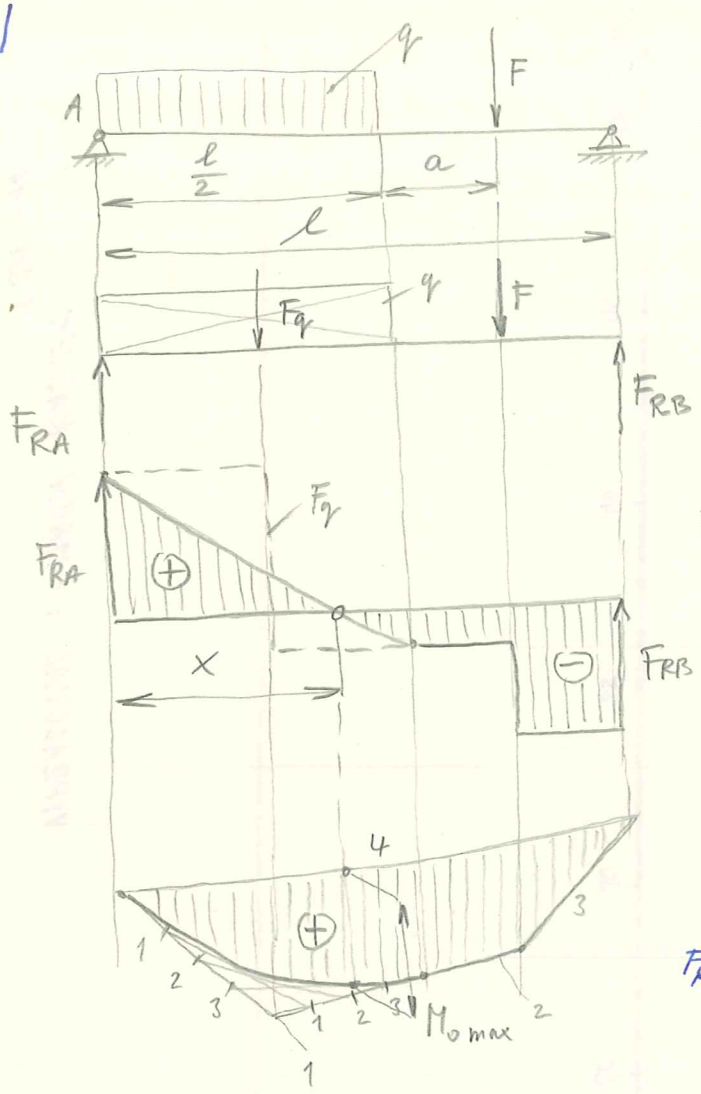
$F_{RB} = \frac{F_q}{2} = 200\,N$

$F_{RA} = F_q - F_{RB} = 400 - 200 = \underline{\underline{200\,N}}$

$M_{o\,max} = F_{RA} \cdot \frac{l}{2} - \frac{F_q}{2} \cdot \frac{l}{4} = \frac{F_q}{2}\frac{l}{2} - \frac{F_q}{2}\frac{l}{4}$

$= \frac{q l^2}{8} = \frac{400 \cdot 1^2}{8} = \underline{\underline{50\,Nm}}$

4.129



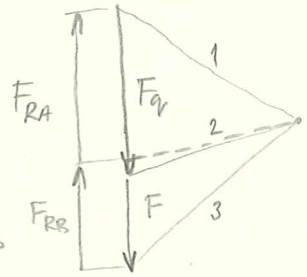
$q = 4500\,N\,m^{-1}$

$\frac{l}{2} = 2\,m$

$a = 1\,m$

$F = 5000\,N$

$x_M = ?$



$F_q = q\frac{l}{2} = 4500 \cdot 2 = \underline{9000\,N}$

$\Sigma F_j = 0 \cdots F_{RA} - F_q - F + F_{RB} = 0$

$\Sigma M_A = 0 \cdots -F_q \frac{l}{4} - F\left(\frac{l}{2}+a\right) + F_{RB} \cdot l = 0$

$F_{RB} = \frac{F_q \frac{l}{4} + F\left(\frac{l}{2}+a\right)}{l} = \frac{9000 \cdot 1 + 5000\,(2+1)}{4} = \underline{6000\,N}$

$F_{RA} = F_q + F - F_{RB} = 9000 + 5000 - 6000 = \underline{8000\,N}$

$F_t = F_{RA} - qx \quad \ldots \quad F_t = 0$

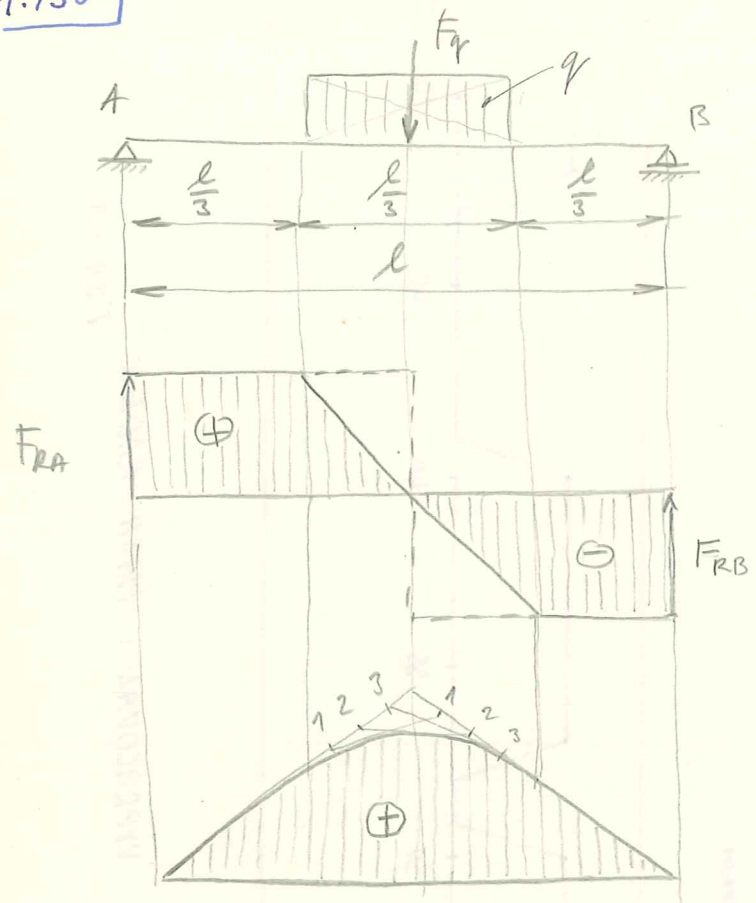
$F_{RA} = qx$

$x = \frac{F_{RA}}{q} = \frac{6000}{4500} = \underline{1{,}777\,m}$

$M_{o\,max} = F_{RA} \cdot x - F_x \frac{x}{2} = 8000 \cdot 1{,}777 - q\frac{x^2}{2} =$

$= 8000 \cdot 1{,}777 - 4500\,\frac{1{,}777^2}{2} = \underline{7111{,}2\,Nm}$

4.130



Zadano: $q, \frac{l}{3}, l$

$F_q = q\frac{l}{3}$

$F_{RA} = F_{RB} = \frac{F_q}{2} = q\frac{l}{6}$

$F_t = 0$ za M_{omax}

$x = \frac{l}{2}$

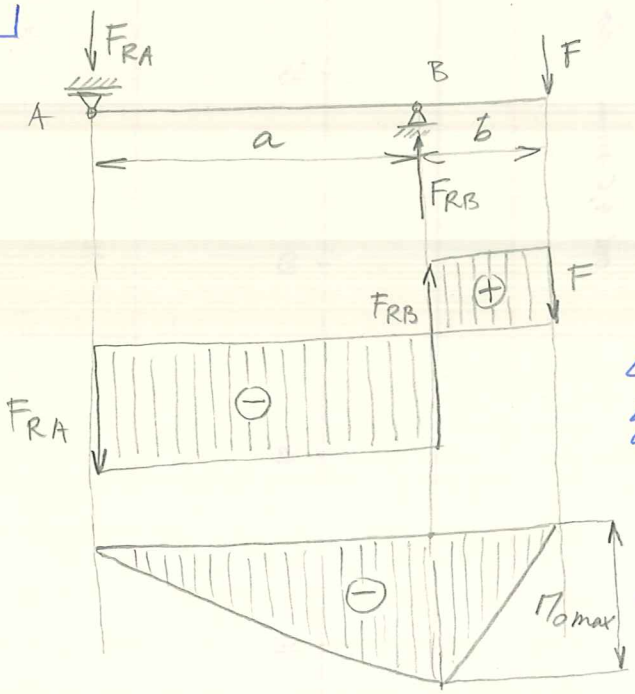
$M_{omax} = F_{RA}\cdot\left(\frac{l}{3}+\frac{l}{6}\right) - q\frac{l}{6}\cdot\frac{l}{12} =$

$= q\frac{l}{6}\frac{l}{2} - q\frac{l}{6}\cdot\frac{l}{12} =$

$= ql^2\left(\frac{1}{12}-\frac{1}{72}\right) - ql^2\frac{5}{72}$

$M_{omax} = \frac{5}{72}ql^2$

4.131



$F = 6000\,N$

$a = 1\,m$

$b = 0{,}5\,m$

Graf. anal.

$M_{omax} = ?$

$\sum F_y = 0 \ldots\ldots -F_{RA} + F_{RB} - F = 0$

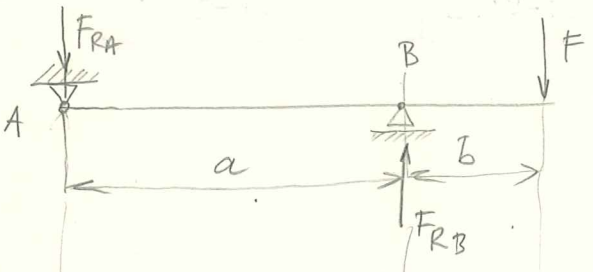
$\sum M_A = 0 \ldots\ldots F_{RB}\cdot a - F(a+l) = 0$

$F_{RB} = F\frac{a+l}{a}$

$F_{RA} = F_{RB} - F = F\frac{a+l}{a} - F = F\left(\frac{a+l}{a} - 1\right)$

$M_{omax} = -F\cdot b$

4.181



$F = 6000\,N$

$a = 1\,m$

$b = 0{,}5\,m$

Graf. anal.

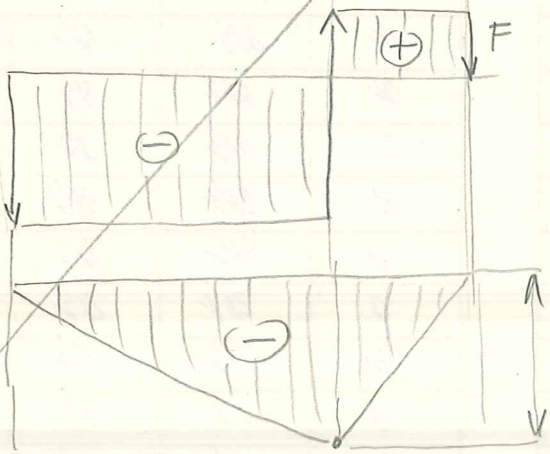
$M_{o\,max} = ?$

$\Sigma F_y = 0 \dots -F_{RA} + F_{RB} - F = 0$

$\Sigma M_A = 0 \dots F_{RB} a - F(a+l) = 0$

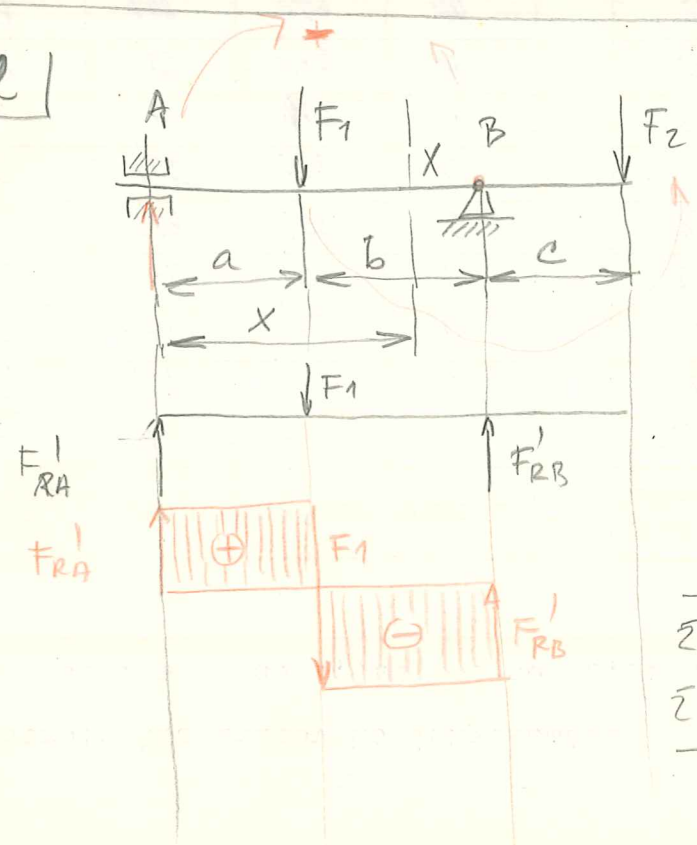
$$F_{RB} = \frac{F(a+l)}{a}$$

$$F_{RA} = F_{RB} - F = F\frac{a+l}{a} - F = F\left(\frac{a+l}{a} - 1\right)$$



$M_{o\,max} = -F \cdot b$

4.182



$F_1 = 10\,000\,N$

$F_2 = 6000\,N$

$a = b = c = 1{,}5\,m$

Anal. superpozice

a graf. F_z

M_o

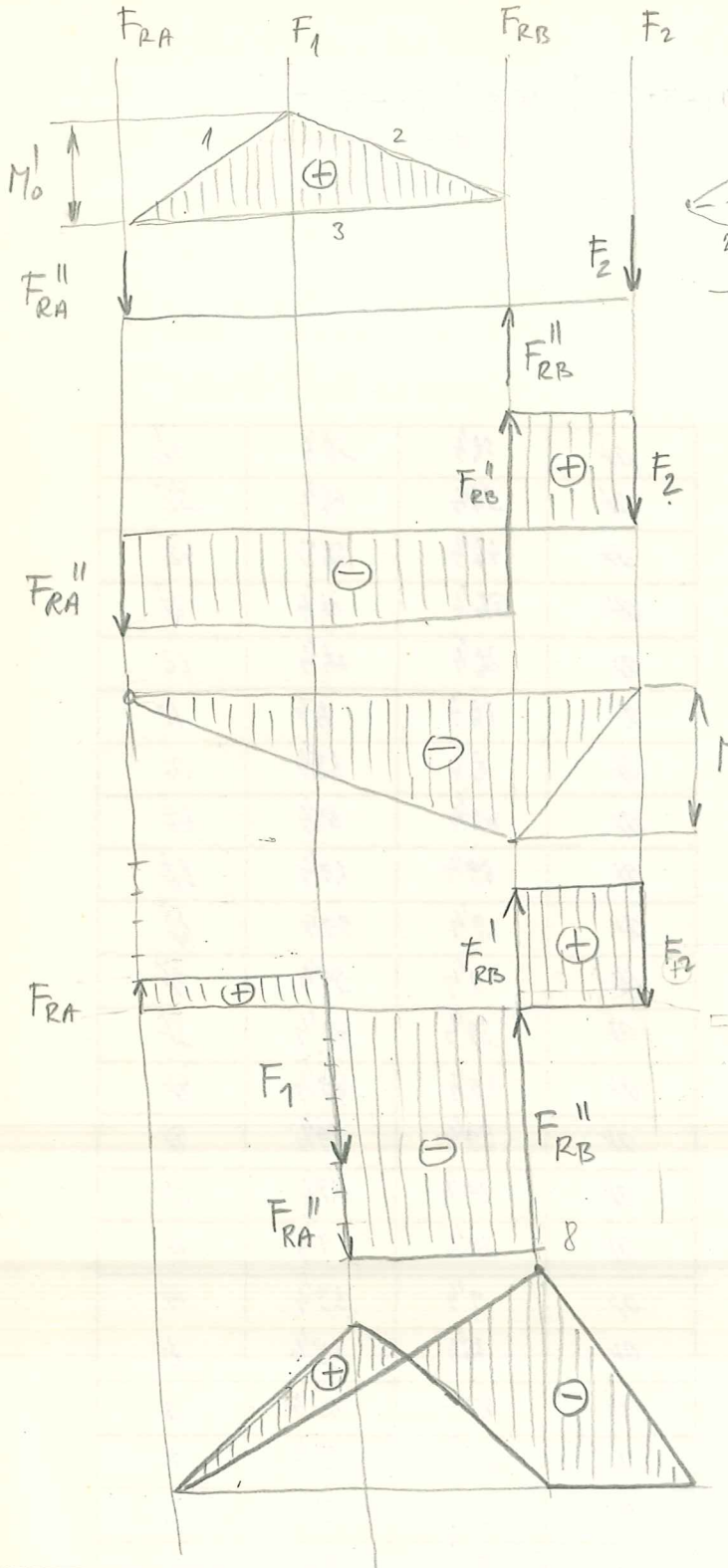
$M_{o\,max} = ?$

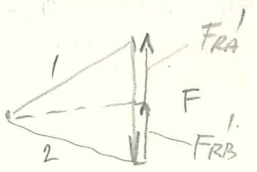
$\Sigma F_y = 0 \dots F'_{RA} - F_1 + F'_{RB} = 0$

$\Sigma M_A = 0 \dots -F_1 a + F'_{RB} \cdot (a+b) = 0$

$$F'_{RB} = \frac{F_1 a}{a+b} = \frac{10 \cdot 10^3 \cdot 1{,}5}{3} = 5 \cdot 10^3\,N$$

$$F'_{RA} = F_1 - F'_{RB} = 10 \cdot 10^3 - 5 \cdot 10^3 = 5 \cdot 10^3\,N$$





$M_0' = F_{RA}' \cdot a = 5.10^3 \cdot 1{,}5 = 7{,}25.10^3\,Nm$

$\Sigma F_y = 0 \dots -F_{RA}'' + F_{RB}'' - F_2 = 0$

$\Sigma M_B = 0 \dots F_{RA}''(a+b) - F \cdot c = 0$

$F_{RA}'' = F_2 \frac{c}{a+b} = 6.10^3 \frac{1{,}5}{3} = 3.10^3\,N$

$F_{RB}'' = F_{RA}'' + F_2 = 3.10^3 + 6.10^3 = 9.10^3\,N$

$M_0'' = -F_2 \cdot c = -6.10^3 \cdot 1{,}5 = -9.10^3\,Nm$

$F_A = F_{RA}' - F_{RA}'' = 5.10^3 - 3.10^3 = 2.10^3\,N$

$F_B = F_{RB}' + F_{RB}'' = 5.10^3 + 9.10^3 = 14.10^3\,N$

$M_{0max} = M_0'' = -9.10^3\,Nm$

$F_A = 2000$

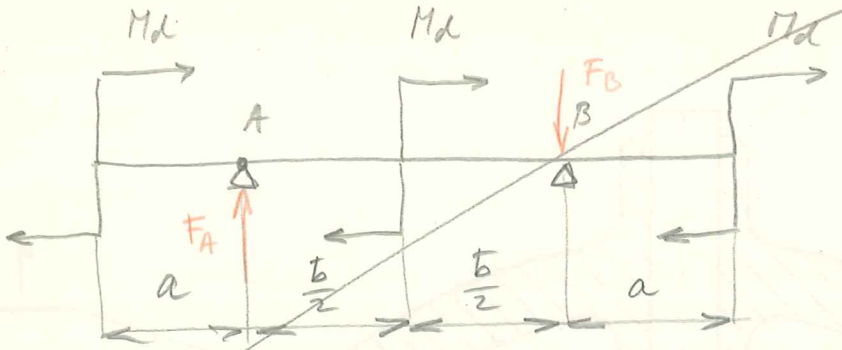
$F_B = 14000$

$M_0 = -3000\ Nm$

Kontrola podatka: M_{max}:

$M_{0max} = F_{RA}(a+b) + F_1 \cdot b = -2000 \cdot 3 + 10\,000 \cdot 1{,}5 = -6000 + 15\,000 = 9\,000\,Nm$

Zadatak

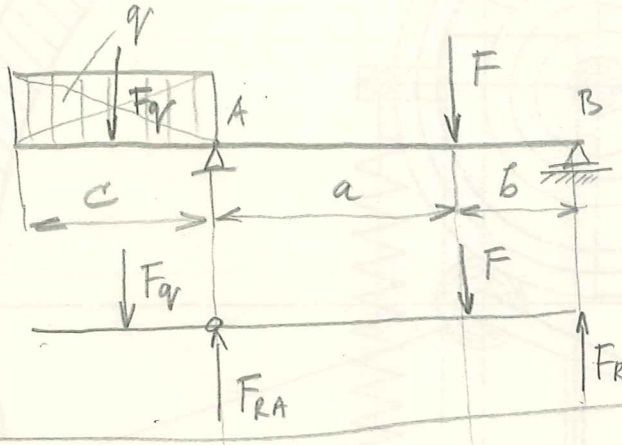


$\Sigma F_x = 0$ ——— 0

$\Sigma F_y = 0$ ——— 0

$\Sigma M_A = 0$

4.134 |



$F = 10\,000\ N$

$q = 8000\ N\,m^{-1}$

$a = 2{,}5\ m$

$b = 1{,}5\ m$

$c = ?$

$M_A = M_F$

$M_F = F_{RB} \cdot b$ | $M_A = F_q \cdot \frac{c}{2}$ | $F_{RB} \cdot b = F_q \frac{c}{2}$

$\Sigma M_A = 0 \dots F_{RB}(a+b) - F.a + F_q \frac{c}{2} = 0$

$F_{RB} = \frac{F.a - F_q \frac{c}{2}}{a+b}$

$$\frac{F.a - q\frac{c^2}{2}}{a+b} \cdot b = q\frac{c^2}{2} \quad / \cdot (a+b)$$

$$\left(F.a - q\frac{c^2}{2}\right) b = (a+b) q \frac{c^2}{2}$$

$$Fab - bq\frac{c^2}{2} = aq\frac{c^2}{2} + bq\frac{c^2}{2} \quad / \cdot 2$$

$$2Fab - bqc^2 = 2aqc^2 + bqc^2$$

$$2Fab = 2aqc^2 + 2bqc^2$$

$$c^2(aq + bq) = Fab$$

$$c = \sqrt{\frac{Fab}{q(a+b)}} = \sqrt{\frac{10\,000 \cdot 2{,}5 \cdot 1{,}5}{8000\,(2{,}5+1{,}5)}} =$$

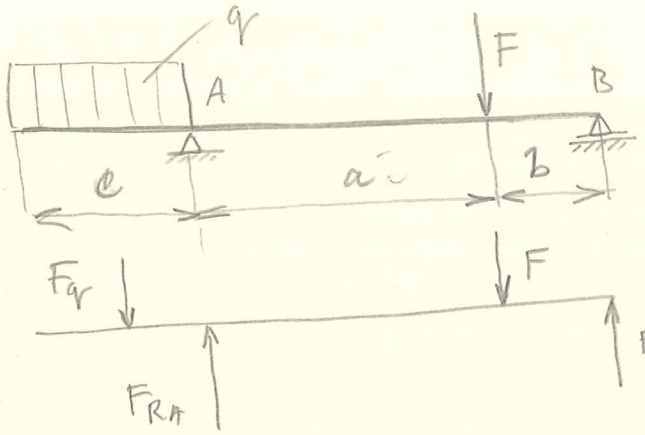
$$= \sqrt{1{,}171875} = 1{,}0825\ m$$

$c = 1{,}306\ m$

6822544

4.134

$c = 1{,}306\ m$



$F = 10\,000\ N$

$q = 8000\ N\,m^{-1}$

$a = 2{,}5\ m$

$b = 1{,}5\ m$

$M_{OF} = M_{OA}$

$c = ?$

$$F_q \cdot \frac{c}{2} = F_{RB} \cdot b$$

$\sum F_x = 0$

$\sum F_y = 0$ $\quad\dots\quad$ $F_q + F_{RA} + F - F_{RB} = 0$

$\sum M_A = 0$ $\quad\dots\quad$ $F_q \cdot \frac{c}{2} - F \cdot a + F_{RB}(a+b) = 0$

$q c - F_{RA} + F - F_{RB} = 0$

$q \frac{c^2}{2} - F \cdot a + F_{RB}(a+b) = 0$

$$q \cdot \frac{c^2}{2} = F_{RB} \cdot b$$

$F_{RB} \cdot b - F a + F_{RB} \cdot a + F_{RB} \cdot b = 0$

$F_{RB} \cdot a + 2 F_{RB} \cdot b - F a = 0$

$q c - F_{RA} + F - F_{RB} = 0$

$F_{RB}(a+2b) - F a = 0$

$$F_{RB} = \frac{F a}{a+2b}$$

$$q \frac{c^2}{2} = \frac{F a b}{a+2b}$$

$$c = \sqrt{\frac{2 F a b}{q(a+2b)}} = \sqrt{\frac{2 \cdot 10\,000 \cdot 2{,}5 \cdot 1{,}5}{8000 \cdot (2{,}5+3)}} =$$

$$= 1{,}30558\ m$$

4.139 $M_F = F_{RB} \cdot b$

$M_A = F_q \frac{c}{2}$

$F_{RB} \cdot b = F_q \frac{c}{2}$

$\Sigma M_A = 0 \quad F_{RB}(a+b) - F \cdot a + F_q \frac{c}{2} = 0$

$$F_{RB} = \frac{F \cdot a - F_2 \frac{c}{2}}{a+b}$$

$$\frac{b\left(F \cdot a - q\frac{c^2}{2}\right)}{a+b} = q\frac{c^2}{2} \quad / \cdot b$$

$$Fab - bq\frac{c^2}{2} = q\frac{c^2}{2}a + q\frac{c^2}{2}b$$

$$Fab = q\frac{c^2}{2}a + 2q\frac{c^2}{2}b$$

$$Fab = c^2\left(\frac{qa}{2} + qb\right)$$

$$c = \sqrt{\frac{Fab}{\frac{qa}{2} + qb}} = \sqrt{\frac{Fab}{q\left(\frac{a}{2}+b\right)}} = \sqrt{\frac{10000 \cdot 2{,}5 \cdot 1{,}5}{8000\left(\frac{2{,}5}{2}+1{,}5\right)}}$$

$$= 1{,}30\sqrt{5}\,m$$

4.135 | $W_o = ?$ $\quad d = 80\,mm$

$$W_o = \frac{\pi}{32} d^3 = \frac{\pi}{32} \cdot 80^3 = \underline{\underline{50265{,}48\,mm^3}}$$

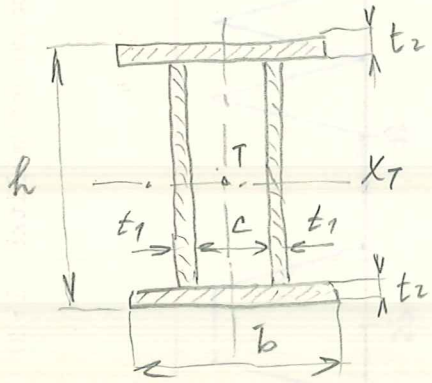
4.136 | $D_1 = 240\,mm$

$D_2 = 220\,mm$

$$W_o = \frac{\frac{\pi}{64}(D_1^4 - D_2^4)}{\frac{D_1}{2}} = \frac{\pi}{32} \cdot \frac{240^4 - 220^4}{240} = \underline{\underline{398916{,}8\,mm^3}}$$

4.137 | ne

4.138 | Vypočtěte modul t ose x_T (př. 4.91)



$J_{xCT} = 51136 \cdot 10^4\,mm^4$

$h = 400\,mm$

$e = e_1 = e_2 = \frac{h}{2} = 200\,mm$

$$W_{o1,2} = \frac{J_{xCT}}{e} = \frac{51136 \cdot 10^4}{200} = \underline{\underline{255{,}68 \cdot 10^4\,mm^3}}$$

4.139 | ocel 11 375 – míjivé napětí; $k = 1{,}6$

$\sigma_{k_o} = 0{,}7\,\sigma_{Pt} = 0{,}7 \cdot 370 = 259$ MPa

$\sigma_{D_o} = \frac{\sigma_{k_o}}{k} = \frac{259}{1{,}6} = 161{,}8$ MPa $\qquad c_{\underline{\underline{II}}} = 0{,}85$

$\sigma_{D_o\underline{\underline{II}}} = c_{\underline{\underline{II}}}\,\sigma_{D_o} = 0{,}85 \cdot 161{,}8 = \underline{137{,}5\ \text{MPa}}$

4.140 | $\sigma_{D_o} = ?$ pro litinu 42 24 35 pro míjivé zatížení

$k = 4$

$\sigma_{Pt} = 350$ MPa

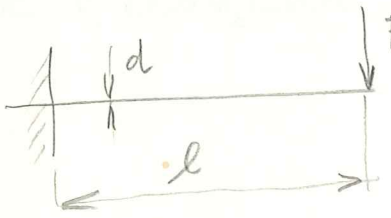
$\sigma_{Dt} = \frac{\sigma_{Pt}}{k} = \frac{350}{4} = 87{,}5$ MPa

$c_{\underline{\underline{II}}} = 0{,}75$

$\sigma_{Dt\underline{\underline{II}}} = c_{\underline{\underline{II}}}\,\sigma_{Dt} = 0{,}75 \cdot 87{,}5 = \underline{65{,}6\ \text{MPa}}$

$\sigma_{D_o\underline{\underline{II}}} = 0{,}76\,\sigma_{Dt\underline{\underline{II}}} = 0{,}7 \cdot 65{,}6 = \underline{45{,}8\ \text{MPa}}$

4.141 |



$d = 12$ mm $\qquad F = ?$ míjivá a rázová

$l = 1$ m

ocel 11 375

$k = 1{,}6$

$c_{\underline{\underline{II}}} = 0{,}85$

ráz = 0,65 %

$\sigma_{D_o\underline{\underline{II}}} = c_{\underline{\underline{II}}}\,\frac{0{,}7\,\sigma_{Pt}}{k} = 0{,}85\,\frac{0{,}7 \cdot 370}{1{,}6} = 137{,}6$ MPa

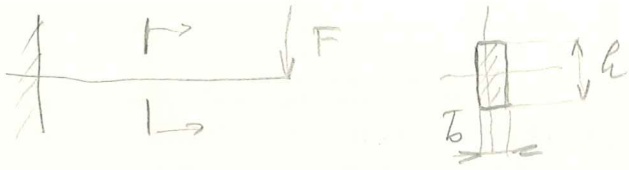
$W_o = \frac{\pi}{32} d^3 = \frac{\pi}{32} \cdot 12^3 = 169{,}64\ \text{mm}^3$

$\sigma_{D_o\underline{\underline{II}}R} = 0{,}65\,\sigma_{D_o\underline{\underline{II}}} = 0{,}65 \cdot 137{,}6 \doteq \underline{90\ \text{MPa}}$

$M_o = W_o \cdot \sigma_{D_o\underline{\underline{II}}R} = 169{,}64 \cdot 90 = 15\,264$ N mm

4.142 Stejné zadání jako u 4.141 tzn. $S_0 = S_\square$

při poměru $h:b = 2:1$



$$S_0 = \frac{\pi d^2}{4} = \frac{\pi \cdot 12^2}{4} = 113{,}097\ mm^2$$

$$S_\square = b \cdot h = 2b^2 \Rightarrow b = \sqrt{\frac{S}{2}} = \underline{7{,}5\ mm}$$

$$h = 2b \qquad h = 2 \cdot 7{,}5 = \underline{15\ mm}$$

$$W_{o\square} = \frac{b h^2}{6} = \frac{7{,}5 \cdot 15^2}{6} = \underline{281{,}25\ mm^3}$$

$$M_o = \sigma_{DO\pi R} \, W_{o\square} = 90 \cdot 281{,}25 = 25312{,}5\ Nmm$$

$$F = \frac{M_o}{l} = \frac{25312{,}5}{1000} = \underline{25{,}31\ N}$$

4.143 Určit S, α_{max}, y_{max} a) nosník z úlohy 4.141

b) nosník z úlohy 4.142

Pro modul pružnosti $E = 2{,}06 \cdot 10^5\ MPa$

a) $$S = \frac{EJ}{M} = \frac{2{,}06 \cdot 10^5 \cdot 1017{,}87}{15264} =$$

$$J_{x\odot} = \frac{\pi d^4}{64} = \frac{\pi \cdot 12^4}{64} = \underline{1017{,}87\ mm^4}$$

$$= 13736{,}9\ mm = \underline{13{,}73\ m}$$

$$\alpha_{max} = \frac{F l^2}{2EJ} = \frac{15{,}26 \cdot 1000^2}{2 \cdot 2{,}06 \cdot 10^5 \cdot 1017{,}87} = \underline{0{,}03638\ rad}$$

$$y_{max} = \frac{F l^3}{3EJ} = \frac{15{,}26 \cdot 1000^3}{3 \cdot 2{,}06 \cdot 10^5 \cdot 1017{,}87} = \underline{24{,}25\ mm}$$

b) $$S = \frac{EJ}{M} =$$

$$J_x = \frac{1}{12} b h^3 = \frac{7{,}5 \cdot 15^3}{12} = \underline{2109{,}3\ mm^4}$$

$$= \frac{2{,}08 \cdot 10^5 \cdot 2109{,}3}{25312{,}5} = 17332\ mm = \underline{17{,}33\ m}$$

$$\alpha_{max} = \frac{F l^2}{2EJ} = \frac{15{,}26 \cdot 1000^2}{2 \cdot 2{,}08 \cdot 10^5 \cdot 2109{,}3} = \underline{0{,}017379\ rad}$$

4.144 | Nosník vetknutý/spoj. zatížený $q = 4000\ N m^{-1}$
$l = 0{,}8\ m$ · Profil nosníku $b = 26\ mm$; $h = 52\ mm$
$E = 2{,}1 \cdot 10^5\ MPa$

$$J = \frac{b h^3}{12} = \frac{26 \cdot 52^3}{12} = 304650{,}6\ mm^4$$

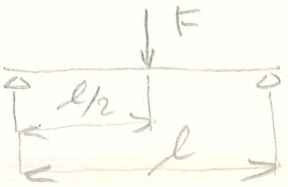
$$M = F_q \cdot \frac{l}{2} = q \frac{l^2}{2} = 4000 \frac{0{,}8^2}{2} = 1280\ Nm$$

$$f = \frac{EJ}{M} = \frac{2{,}1 \cdot 10^5 \cdot 304650{,}6}{1280 \cdot 10^3} = 49981{,}7\ mm = 49{,}981\ m$$

$$\alpha = \frac{q l^3}{6EJ} = \frac{4 \cdot 800^3}{6 \cdot 2{,}1 \cdot 10^5 \cdot 304650{,}6} = 5{,}335 \cdot 10^{-3}\ rad$$

$$y = \frac{q l^4}{8EJ} = \frac{4 \cdot 800^4}{8 \cdot 2{,}1 \cdot 10^5 \cdot 304650{,}6} = 3{,}201\ mm$$

4.145 |



$\alpha_{max} = ?$ $\quad y_{max} = ?$

$F = 1000\ N$ $\quad l = 2\ m$ $\quad E = 2{,}1 \cdot 10^5\ MPa$

$b = 20\ mm$
$h = 40\ mm$ ocel 11500

Kontrola, zda je def. v oblasti pružné!

$$y_{max} = \frac{F l^3}{48EJ} = \frac{1000 \cdot 2000^3}{48 \cdot 2{,}1 \cdot 10^5 \cdot 106666{,}6} = 7{,}44\ mm$$

$$\alpha_{max} = \frac{F l^2}{16EJ} = \frac{1000 \cdot 2000^2}{16 \cdot 2{,}1 \cdot 10^5 \cdot 106666{,}6} = 0{,}01116\ rad$$

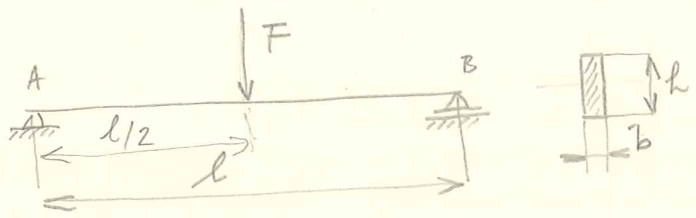
$$J = \frac{b h^3}{12} = \frac{20 \cdot 40^3}{12} = 106666{,}6\ mm^4$$

$$\sigma_o = \frac{M_{o\,max}}{W_o} = \frac{F_{RA} \cdot \frac{l}{2}}{\frac{1}{6} b h^2} = \frac{6 \cdot 500 \cdot 1000}{20 \cdot 40^2} = 93{,}75\ MPa$$

$$\sigma_{Ko} = 0{,}76\, \sigma_{Pt} = 0{,}7 \cdot 500 = 350\ MPa$$

4. 145

2A



$\alpha_{max} = ?$
$y_{max} = ?$
$\sigma_0 = ?$
11 500

Je def. v pružné oblasti?

$F = 1000\,N$ $\quad b = 20\,mm$
$l = 2000\,mm$ $\quad h = 40\,mm$ $\quad E = 2,1 \cdot 10^5\,MPa$

$$\alpha_{max} = \alpha_A = \alpha_B = \frac{S_T}{EJ} = \frac{Fl^2}{16EJ} = \frac{1000 \cdot 2000^2}{16 \cdot 2 \cdot 10^5 \cdot 106666,6} = 1171,8 \cdot 10^{-5}$$
$$= 0,01718\,rad$$

$$J_x = \frac{1}{12} b h^3 = \frac{1}{12} 20 \cdot 40^3 = 106\,666,6\,mm^4$$

$$y_{max} = \frac{Fl^3}{48EJ} = \frac{1000 \cdot 2000^3}{48 \cdot 2,1 \cdot 10^5 \cdot 106\,666,6} = 781250 \cdot 10^{-5} = 7,812\,mm$$

$$\sigma_0 = \frac{M_{0max}}{W_0} = \frac{F_{RA} \cdot \frac{l}{2}}{W_0} = \frac{500 \cdot 1000}{5333,3} = 93,7\,MPa \quad \Big| \; F_{RA} = F_{RB} = \frac{F}{2} = 500\,N$$

$$W_0 = \frac{b h^2}{6} = \frac{20 \cdot 40^2}{6} = 5333,3\,mm^3$$

$$\sigma_{K0} = 0,7\,\sigma_{Pt} = 0,7 \cdot 500 = 350\,MPa$$

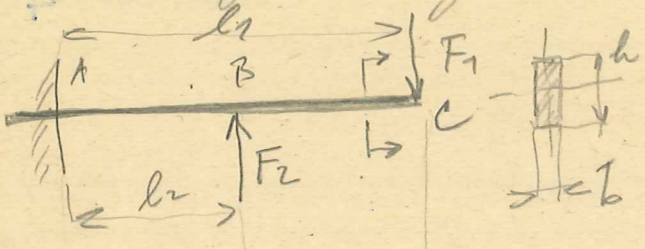
$$\sigma_0 < \sigma_{K0}$$

DEFORMACE JE V PRUŽNÉ OBLASTI

4.146) M_d – u vetknutého nosníka. Jedná sa o ohyb síl.

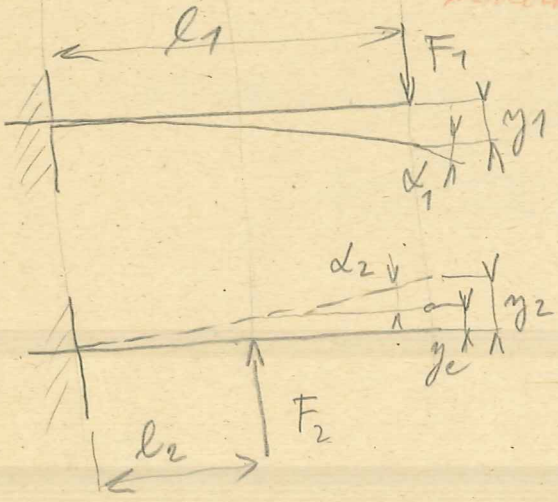
Ohybová čiara je kružnica, pretože ide o čistý ohyb (ohyb prameňových síl)

4.147) Drevený vetknutý nosník



$b = 100\,mm$
$h = 140\,mm$
$F_1 = 3000\,N$
$F_2 = 5000\,N$
$l_1 = 1{,}5\,m$
$l_2 = 0{,}6\,m$
$E = 9 \cdot 10^3\,MPa$
$\alpha_{max} = ?$
$y_{max} = ?$

Rieši analyticky – Grob riešiť nedotlačí!



Superpozícia

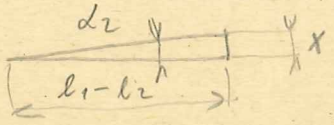
$$J = \frac{bh^3}{12} = \frac{100 \cdot 140^3}{12} = 22866666{,}6\,mm^4$$

$$\alpha_1 = \frac{F_1 l^2}{2EJ} = \frac{3000 \cdot 1500^2}{2 \cdot 9{,}10^3 \cdot 22866666} = 0{,}016399\,rad$$

$$y_1 = \frac{F_1 l^3}{3EJ} = \frac{3000 \cdot 1500^3}{3 \cdot 9 \cdot 10^3 \cdot 22866666} = 16{,}399\,mm$$

$$\alpha_2 = \frac{F_2 l_2^2}{2EJ} = \frac{5000 \cdot 600^2}{2 \cdot 9{,}10^3 \cdot 22866666} = 4{,}37317 \cdot 10^{-3}\,rad$$

$$y_c = \frac{F_2 l_2^3}{3EJ} = \frac{5000 \cdot 600^3}{2 \cdot 9 \cdot 10^3 \cdot 22866666} = 2{,}6239\,mm$$

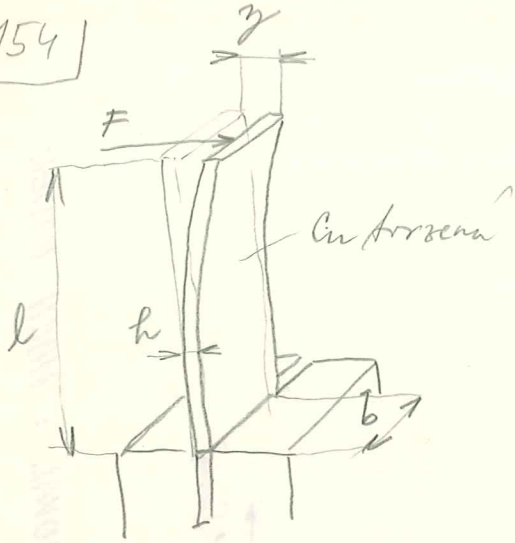


$$x = (l_1 - l_2) \cdot \operatorname{tg} \alpha_2 = 900 \cdot 4{,}37319 \cdot 10^{-3} = 3{,}9358\,mm$$

$$y_2 = y_c + x = 2{,}6239 + 3{,}9358 = 6{,}5597\,mm$$

$$\alpha = \alpha_1 - \alpha_2 = 0{,}016399 - 0{,}0043737 = 0{,}0120253\,rad$$

4.154



$y = 0{,}25\ mm$

$b = 22\ mm$

$F = 17\ N$

$l = 34\ mm$

$E = 1{,}1 \cdot 10^5\ MPa$

$\sigma_{D_0} = 60\ MPa$

$h = ?$

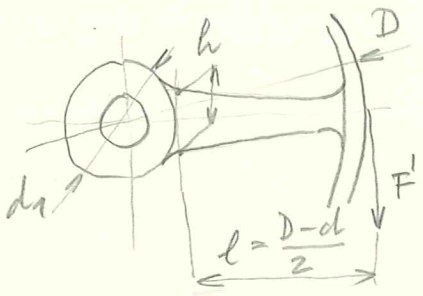
$$y = \frac{Fl^3}{3EJ} \Rightarrow J = \frac{Fl^3}{3Ey} = \frac{17 \cdot 34^3}{3 \cdot 1{,}1 \cdot 10^5 \cdot 0{,}25} = 8{,}099\ mm^4$$

$$J = \frac{bh^3}{12} \Rightarrow h = \sqrt[3]{\frac{12J}{b}} = \sqrt[3]{\frac{12 \cdot 8{,}099}{22}} = \underline{1{,}64\ mm}$$

$$\sigma_0 = \frac{M_0}{W_0} = \frac{F \cdot l}{\frac{bh^2}{6}} = \frac{6Fl}{bh^2} = \frac{6 \cdot 17 \cdot 34}{22 \cdot 1{,}64^2} = \underline{58{,}6\ MPa}$$

$\sigma_0 < \sigma_{D_0}$

4.155



$D = 500\ mm$ $\quad d_1 = 100\ mm$

$M_k = 656\ Nm$ $\quad b = 29\ mm$

Čtyřramen $\quad h = 58\ mm$

Šedá litina

$\sigma_{D_0} = 30\ MPa$

Kontrola pevnosti

$$F' = \frac{M_k}{i \cdot \frac{D}{2}} = \frac{2M_k}{iD} = \frac{3 \cdot 2 \cdot 656 \cdot 10^3}{6 \cdot 500} = 1311,9\ N$$

$$M_o = F' \cdot l = F' \frac{D-d}{2} = 1311,9 \cdot \frac{500-100}{2} = 262380\ Nmm$$

$$\sigma_o = \frac{M_o}{W_o} = \frac{M_o}{\frac{\pi}{32} b h^2} = \frac{32 \cdot 262380}{\pi \cdot 29 \cdot 58^2} = 27,39\ MPa$$

$\sigma_o < \sigma_{D_o}$

4.156 Řemenice $D = 960\ mm$; $d_1 = 130\ mm$ obvodová síla $F = 1320\ N$

eliptické rameno $b : h = 1 : 2$; $\sigma_{D_o} = 30\ MPa$

$i = 6$

$$M_o = \frac{3F}{i} l = \frac{3F}{i} \frac{D-d}{2} = \frac{3 \cdot 1320}{6} \frac{960-130}{2} = 273900\ Nmm$$

$$W_o = \frac{M_o}{\sigma_{D_o}} = \frac{273900}{30} = 9130\ mm^3$$

$b = 30$
$h = 60$

$$W_o = \frac{\pi}{32} b h^2 = \frac{\pi}{32} \frac{h}{2} h^2 \Rightarrow h = \sqrt[3]{\frac{64 W_o}{\pi}} =$$

$b = \frac{h}{2}$

$$= \sqrt[3]{\frac{64 \cdot 9130}{\pi}} = 57,08\ mm$$

$h \doteq 60\ mm$

$b = 30\ mm$

4,157

$D = 1450\,mm$, $d = 200\,mm$

$P = 38\,kW$; $\omega = 8{,}38\,s^{-1}$

$i = 6$; $b:h = 1:2{,}5$ $\quad \sigma_{Do} = 30\,MPa$

$$M_k = \frac{P}{\omega} = \frac{38.10^3}{8{,}38} = 4534{,}6\,Nm$$

$$F' = 3\,\frac{M_k}{i\cdot\frac{D}{2}} = \frac{3.4534{,}6.10^3}{6.\frac{1450}{2}} = 3127{,}3\,N$$

$$M_o = F'l = F'\,\frac{D-d}{2} = 3127{,}3\,\frac{1450-200}{2} = 1954562{,}5\,Nmm$$

$$W_o = \frac{M_o}{\sigma_{Do}} = \frac{1954562{,}5}{30} = 65152{,}08\,mm^3$$

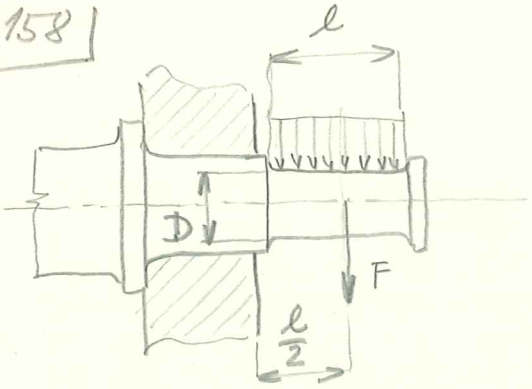
$$W_o = \frac{\pi}{32}\,b\,h^2 = \frac{\pi}{32}\,\frac{h}{2{,}5}\,h^2 \Rightarrow h = \sqrt[3]{\frac{32.2{,}5\,W_o}{\pi}} = \sqrt[3]{\frac{32.2{,}5.65152{,}08}{\pi}} =$$

$$\doteq 118{,}4\,mm$$

$$h = 120\,mm$$

$$b = 48\,mm$$

4. 158



$F = 8000\,N$ – střídavě se měnící

$\sigma_{cp} \dots 11500$

$l : D = 1,2$

$p_D = 6\,MPa$

$k = 3$

$\frac{l}{D} = 1,2 \dots l = 1,2D$

$F = S p_D = l D p_D = 1,2 D . D . p_D \Rightarrow D = \sqrt{\frac{F}{1,2 p_D}} = \sqrt{\frac{8000}{1,2.6}} = 33,3$

$D = 34\,mm$

$l = 1,2D = 1,2.34 = 40,8\,mm$

$M_o = F \frac{l}{2} = 8000 \frac{40,8}{2} = 163200\,Nmm$

$W_o = \frac{\pi}{32} D^3 = \frac{\pi}{32} \cdot 34^3 = 3858,68\,mm^3$

$\sigma_o = \frac{M_o}{W_o} = \frac{163\,200}{3858,68} = 42,29\,MPa$

$\sigma_{Do\,III} = c_{III} \frac{0,7 \sigma_{pt}}{k} = 0,65 \frac{0,7.500}{3} = 75,83\,MPa$

4.159) obr. 4.84 $F = 100\,000\,N$

$l = 180\,mm$

$D = 120\,mm$ Provedte kontrolu čepu

$\sigma_{D_0} = 55\,MPa$

Otlačení $p = \frac{F}{Dl} = \frac{100\,000}{120 \cdot 180} = 4,63\,MPa$

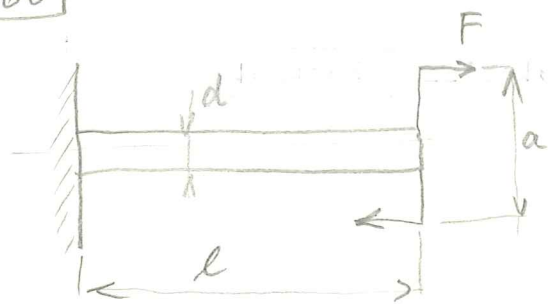
Ohyb $M_o = F \cdot \frac{l}{2} = 100\,000 \cdot \frac{180}{2} = 9\,000\,000\,Nmm$

$W_o = \frac{\pi}{32} D^3 = \frac{\pi}{32} \cdot 120^3 = 169\,646\,mm^3$

$\sigma_o = \frac{M_o}{W_o} = \frac{9\,000\,000}{169\,646} = 53,05\,MPa$

$\sigma_o < \sigma_{D_o}$

4,160



$l = 1\,m$

$F = 800\,N$

$a = 0{,}5\,m$

$d = ?$ $\qquad \sigma_{Do} = 80\,MPa$

$\zeta = ?$ $\qquad E = 2{,}1 \cdot 10^5\,MPa$

$y_{max} = ?$

$$M_o = F \cdot a = 800 \cdot 500 = 400\,000\,N\,mm$$

$$W_o = \frac{M_o}{\sigma_{Do}} = \frac{400\,000}{80} = 5000\,mm^3$$

$$W_o = \frac{\pi}{32} d^3 \Rightarrow d = \sqrt[3]{\frac{32\,W_o}{\pi}} = \sqrt[3]{\frac{32 \cdot 5000}{\pi}} = 37{,}06\,mm$$

$$d = 38\,mm$$

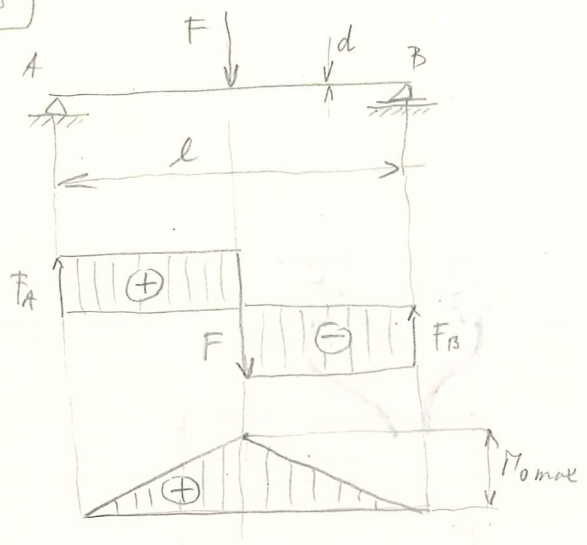
$$J = \frac{\pi}{64} d^4 = \frac{\pi}{64} \cdot 38^4 = 102\,353{,}8\,mm^4$$

$$\zeta = \frac{EJ}{M_o} = \frac{2{,}1 \cdot 10^5 \cdot 102\,353{,}8}{400\,000} = 53\,735{,}7\,mm = 53{,}73\,m$$

$$y = \frac{F l^3}{3EJ} = \frac{400 \cdot 1000^3}{3 \cdot 2{,}1 \cdot 10^5 \cdot 53\,735{,}7} = 11{,}81\,mm$$

$$F = \frac{M_o}{l} = \frac{400\,000}{1000} = 400\,N$$

4.163



$l = 2\,m$

$F = 6000\,N$ 11 340 $E = 2{,}0 \cdot 10^5\,MPa$

$k = 2$

$d = ?$ $y_{max} = ?$

$s = ?$

$$\sigma_{D_o} = \frac{0{,}7\,\sigma_{Pt}}{k} = \frac{0{,}7 \cdot 340}{2} = 119\,MPa$$

$$M_{o\,max} = \frac{F}{2}\,\frac{l}{2} = \frac{6000 \cdot 2000}{4} = 3000\,000\,Nmm$$

$$W_o = \frac{M_{o\,max}}{\sigma_{D_o}} = \frac{3000\,000}{119} = 25\,210\,mm^3$$

$$W_o = \frac{\pi}{32} d^3 \longrightarrow d = \sqrt[3]{\frac{32 W_o}{\pi}} = \sqrt[3]{\frac{32 \cdot 25\,210}{\pi}} = 63{,}56\,mm$$

$$d = 65\,mm$$

$$s = \frac{\pi d^2}{4} = \frac{\pi \cdot 65^2}{4} = 3318{,}3\,mm^2$$

$$y_{max} = \frac{F l^3}{48 E J} = \frac{6000 \cdot 2000^3}{48 \cdot 2{,}0 \cdot 10^5 \cdot 876\,240{,}5} = 5{,}7\,mm$$

$$J = \frac{\pi}{64} d^4 = \frac{\pi}{64} \cdot 65^4 = 876\,240{,}5\,mm^4$$

4.164 | z 4.163 nahradit kd obdélník, $b:h = 1:2$

$b = ?$ $\quad S_\phi = ?$

$h = ?$ $\quad \gamma_{max} = ?$ Porovnejte plochy obou průřezů

$W_o = 25\,210\ mm^3$

$$W_o = \frac{b h^2}{6} = \frac{\frac{h}{2} h^2}{6} - \frac{h^3}{12} \Rightarrow h = \sqrt[3]{12 W_o} = \sqrt[3]{12 \cdot 25\,210} =$$

$$= 67{,}13\ mm$$

$b = \frac{h}{2}$

$\underline{h = 68\ mm}$

$\underline{b = 34\ mm}$

$S_\phi = 6h = 34 \cdot 68 = \underline{2312\ mm^2}$

$$\gamma_{max} = \frac{F l^3}{48 E J} = \frac{6000 \cdot 2000^3}{48 \cdot 2 \cdot 10^5 \cdot 890\,890{,}6} = \underline{5{,}61\ mm}$$

$\gamma = 5$

$$J = \frac{b h^3}{12} = \frac{34 \cdot 68^3}{12} = 890\,890{,}6\ mm^4$$

$$\frac{S_\phi}{S_\phi} = \frac{3318{,}3}{2312} = \underline{1{,}435}$$

4.165 | Nahradte nosník I profilem

$W_o = 25\,210\ mm^3 = 25{,}21\ cm^3$

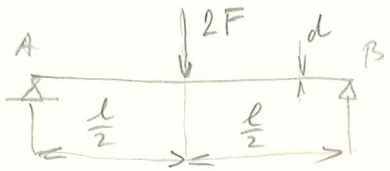
I 100 $\quad W_x = 34{,}2 \cdot 10^3\ mm^3$

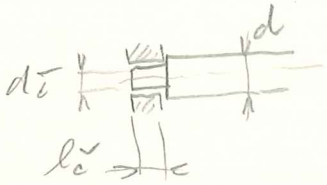
$J_x = 117 \cdot 10^4\ mm^4$

$$\gamma_{max} = \frac{F l^3}{48 E J_x} = \frac{6000 \cdot 2000^3}{48 \cdot 2 \cdot 10^5 \cdot 117 \cdot 10^4} = \underline{4{,}27\ mm}$$

$\underline{S = 1060\ mm^2}$

4.166





$F = 2000\,N$ $\quad E = 2{,}1 \cdot 10^5$

$l = 300\,mm$

čepové jsou o 2 mm menší

11 340, $t = 2$

$y_{max} = ?$ $\quad l_{č} = ?$

$d = ?$ $\quad p_D = 10\,MPa$

$F_A = F_B = 2000\,N$

$$M_0 = F_A \cdot \frac{l}{2} = 2000 \cdot \frac{300}{2} = 300\,000\,Nmm$$

$$\sigma_{DO_{III}} = \frac{0{,}176\,\sigma_{Pt}}{t} \cdot c_{III} = \frac{0{,}17 \cdot 340}{2} \cdot 0{,}65 = \underline{77{,}35\,MPa}$$

$$W_0 = \frac{M_0}{\sigma_{DO_{III}}} = \frac{300\,000}{77{,}35} = 3878{,}4\,mm^3$$

$$W_0 = \frac{\pi}{32} d^3 \longrightarrow d = \sqrt[3]{\frac{32 W_0}{\pi}} = \sqrt[3]{\frac{32 \cdot 3878{,}4}{\pi}} = \underline{34{,}05\,mm}$$

$$\underline{d = 35\,mm}$$

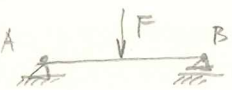
$$y_{max} = \frac{2F \cdot l^3}{48 E J} = \frac{4000 \cdot 300^3}{48 \cdot 2{,}1 \cdot 10^5 \cdot 73661{,}75} = \underline{0{,}145\,mm}$$

$$J = \frac{\pi}{64} d^4 = \frac{\pi}{64} \cdot 35^4 = 73661{,}75$$

$$d_{č} = 35 - 2 = 33\,mm$$

$$F_A = l_{č}\, d_{č} \cdot p_D \Longrightarrow l_{č} = \frac{F_A}{d_{č} \cdot p_D} = \frac{2000}{33 \cdot 10} = \underline{6{,}06\,mm}$$

4,167 $F = ?$ $I\ 140$ $E = 2{,}1 \cdot 10^5\ MPa$



$l = 5{,}2\ m$

$y_{max} = 8\ mm$ $\sigma_0 = ?$

$F = 4820$

$\sigma_0 = 56\ MPa$

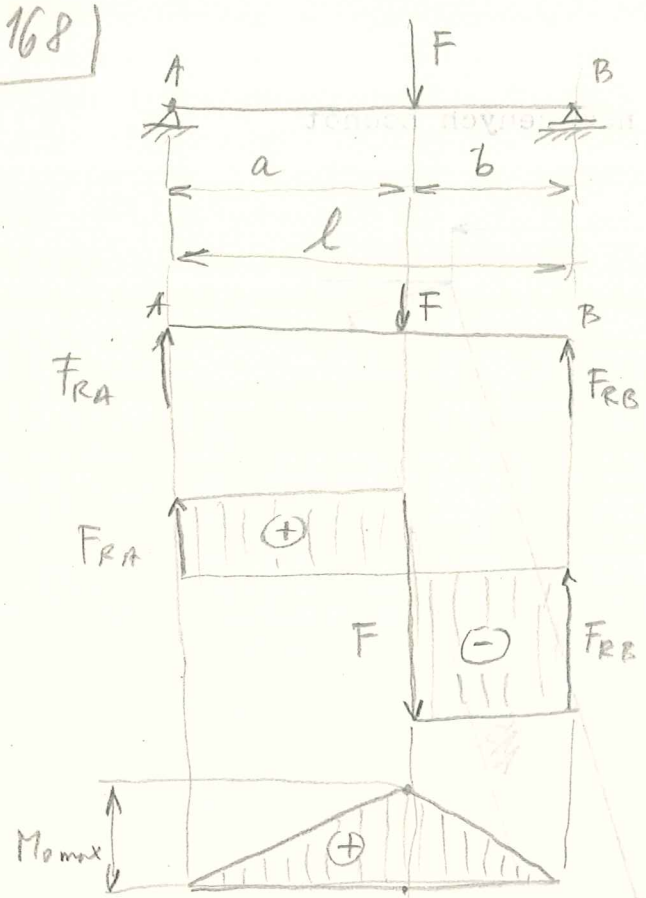
$I\ 140 \dots J_x = 573 \cdot 10^4\ mm^4$; $W_x = 87{,}9 \cdot 10^3\ mm^3$

$$y_{max} = \frac{Fl^3}{48EJ}$$

$$F = \frac{y \cdot 48EJ}{l^3} = \frac{8 \cdot 48 \cdot 2{,}1 \cdot 10^5 \cdot 573 \cdot 10^4}{5200^3} = \underline{3286{,}2\ N}$$

$$\sigma_0 = \frac{M_0}{W_0} = \frac{\frac{F}{2} \cdot \frac{l}{2}}{W_0} = \frac{3286{,}2 \cdot 5200}{4 \cdot 87{,}9 \cdot 10^3} = \underline{52{,}16\ MPa}$$

4.168)



$l = 4,8\,m$ $\quad a = 3\,m$ $\quad b = 1,8\,m$ $\quad F = 6000\,N$

$\sigma_{D_0} = 90\,MPa$ $\quad E = 2,1 \cdot 10^5\,MPa$

$I = ?$

$F_{RA}, F_{RB} = ?$

F_z, M_0

$y_{max} = ?$

$\Sigma F_y = 0 \dots F_{RA} - F + F_{RB} = 0$

$\Sigma M_A = 0 \dots -F \cdot a + F_{RB} \cdot l = 0$

$$F_{RB} = \frac{F \cdot a}{l} = \frac{6000 \cdot 3}{4,8} = 3750\,N$$

$$F_{RA} = F - F_{RB} = 6000 - 3750 = 2250\,N$$

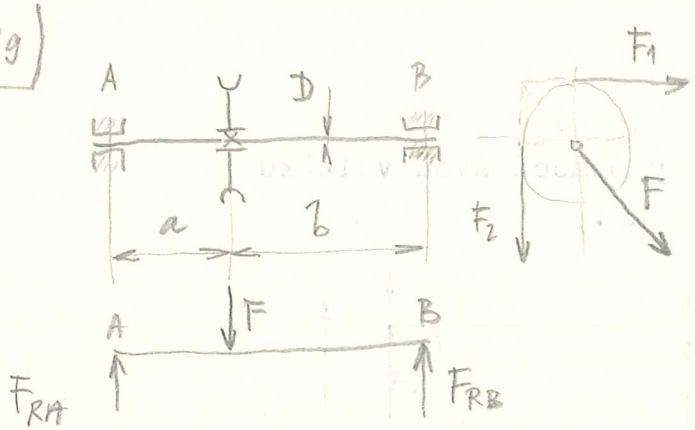
$$M_{0max} = F_{RA} \cdot a = 2250 \cdot 3000 = 675 \cdot 10^4\,N\,mm$$

$$W_0 = \frac{M_{0max}}{\sigma_{D_0}} = \frac{675 \cdot 10^4}{90} = 7,5 \cdot 10^4\,mm^3 = 75 \cdot 10^3\,mm^3$$

I 140 $\dots$ $J_x = 573 \cdot 10^4\,mm^4$

$$y_{max} = \frac{F a^2 b^2}{3 E J l} = \frac{6000 \cdot 3000^2 \cdot 1800^2}{3 \cdot 2,1 \cdot 10^5 \cdot 573 \cdot 10^4 \cdot 4800} = 10,09\,mm$$

4.169)



$F_1 = F_2 = 9200\,N$

$a = 180\,mm$

$b = 380\,mm$

$D = ?$

$y_{max} = ?$

$E = 2{,}1 \cdot 10^5\,MPa$

mírné a rázové zatížení

$t = 2$

11420

$$F = \sqrt{F_1^2 + F_2^2} = \sqrt{9200^2 + 9200^2} = 13010{,}76\,N$$

$$\Sigma F_y = 0 \dots\dots F_{RA} - F + F_{RB} = 0$$

$$\Sigma M_A = 0 \dots\dots -F.a + F_{RB}.(a+b) = 0$$

$$F_{RB} = \frac{F.a}{a+b} = 13010{,}76\,\frac{180}{180+380} = 4182{,}03\,N$$

$$F_{RA} = F - F_{RB} = 13010{,}76 - 4182{,}03 = 8828{,}73\,N$$

$$M_{ohmax} = F_{RA}.a = 8828{,}73 \cdot 180 = 1589171{,}4\,N\,mm$$

$$\sigma_{DOII\,R} = c_{II} \cdot 0{,}65\,\frac{0{,}7\,\sigma_t}{t} = 0{,}85 \cdot 0{,}65\,\frac{0{,}7 \cdot 420}{2} = 81{,}2\,MPa$$

$$W_o = \frac{M_{ohmax}}{\sigma_{DOII\,R}} = \frac{1589171{,}4}{81{,}2} = 19571\,mm^3$$

$$W_o = \frac{\pi D^3}{32} \rightarrow D = \sqrt[3]{\frac{32\,W_o}{\pi}} = \sqrt[3]{\frac{32 \cdot 19571}{\pi}} = 58{,}4\,mm$$

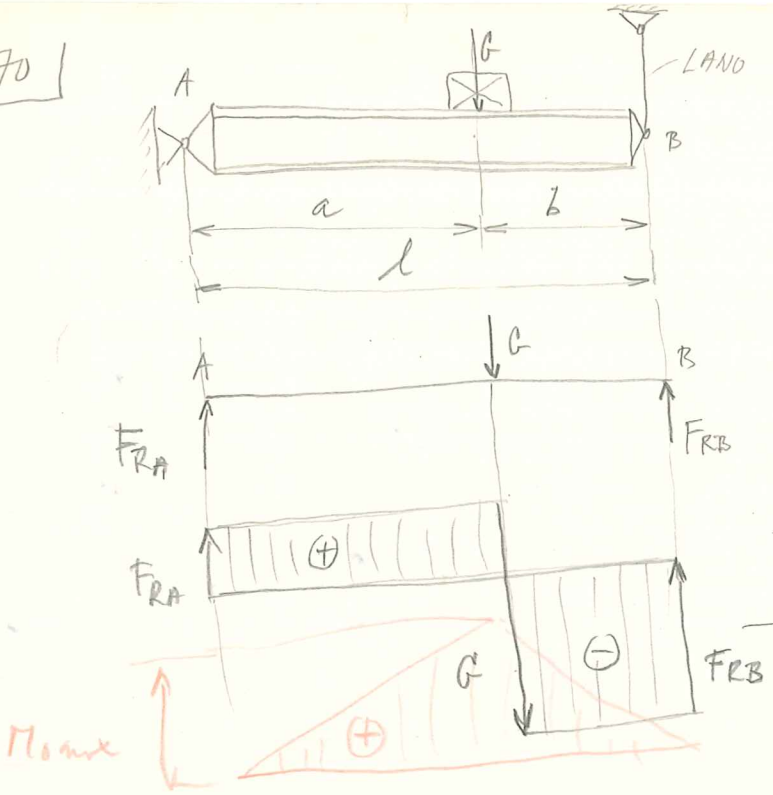
$$d = 60\,mm$$

$$y_{max} = \frac{F\,a^2\,b^2}{3\,E\,J\,l} = \frac{13010{,}76 \cdot 180^2 \cdot 380^2}{3 \cdot 2{,}1 \cdot 10^5 \cdot 636172{,}5 \cdot 560} = 0{,}2712\,mm$$

0,3

$$J = \frac{\pi}{64} D^4 = \frac{\pi}{64} \cdot 60^4 = 636172{,}5\,mm^4$$

4.170



I

$l = 5\,m$

$a = 3\,m$

$G = 20\,000\,N$

F_t, M_o – průběh

$I = ?$

10 452

$t = 2$

$y_{max} = ?$

$\Sigma F_y = 0 \dots\dots F_{RA} - G + F_{RB} = 0$

$\Sigma M_A = 0 \dots\dots -G \cdot a + F_{RB} \cdot l = 0$

$$F_{RB} = \frac{G a}{l} = 20000 \frac{3}{5} = \underline{12000\,N}$$

$$F_{RA} = G - F_{RB} = 20000 - 12000 = \underline{8000\,N}$$

$$M_{o\,max} = F_{RA} \cdot a = 8000 \cdot 3000 = \underline{24 \cdot 10^6\,N\,mm}$$

$$\sigma_{D_o} = \frac{0{,}75 R_e}{k} = \frac{0{,}7 \cdot 450}{2} = \underline{157{,}5\,MPa}$$

$$W_o = \frac{M_{o\,max}}{\sigma_{D_o}} = \frac{24 \cdot 10^6}{157{,}5} = 152380{,}9\,mm^3 = 152{,}38 \cdot 10^3\,mm^3$$

I 180 ... $J_x = 1450 \cdot 10^4\,mm^4$ 16,6

$$y_{max} = \frac{G a^2 b^2}{3 E J l} = \frac{20000 \cdot 3000^2 \cdot 2000^2}{3 \cdot 2{,}1 \cdot 10^5 \cdot 1450 \cdot 10^4 \cdot 5000} = \underline{15{,}76\,mm}$$

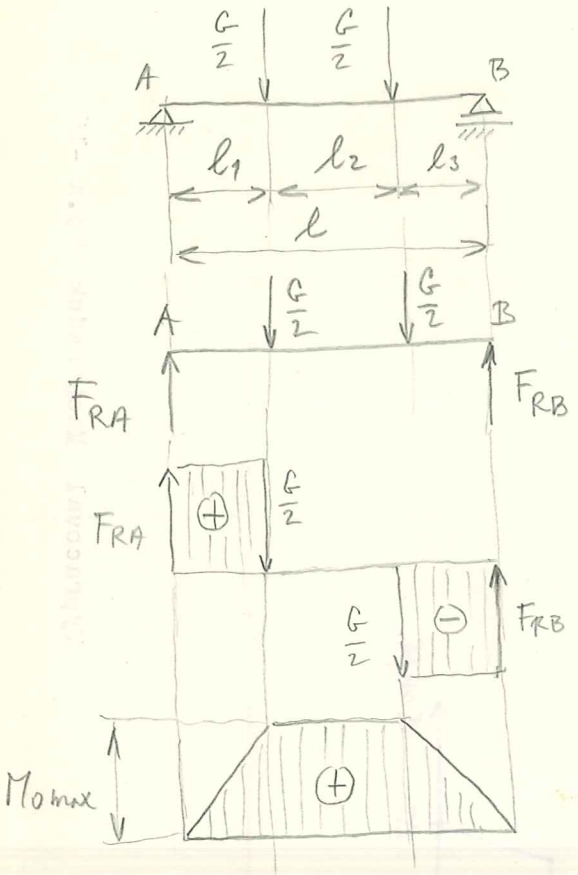
4.171

2 U profily max. síla $G = 68.10^3\ N$

$l = 1{,}7\ m$, $l_1 = 0{,}53\ m$, $l_2 = 0{,}64\ m$

11 552 -- $\sigma_{D_0} = 90\ MPa$ – graf. zjištěte $y_{max} = ?$

$E = 2{,}1.10^5\ MPa$ a velikost profilu „U"



$\Sigma F_y = 0$ ---- $F_{RA} - \frac{G}{2} - \frac{G}{2} + F_{RB} = 0$

$\Sigma M_A = 0$ ---- $-\frac{G}{2} l_1 - \frac{G}{2}(l_1 + l_2) + F_{RB} . l = 0$

$$F_{RB} = \frac{\frac{G}{2}[l_1 + (l_1 + l_2)]}{l} = \frac{34.10^3 (0{,}53 + 0{,}53 + 0{,}64)}{1{,}7}$$

$$= 34.10^3\ N$$

$F_{RA} = 34.10^3\ N$

$M_{o\,max} = F_{RA} . l_1 = 34.10^3 . 530 = 1802.10^4\ Nmm$

$$W_o = \frac{M_{o\,max}}{\sigma_{D_o}} = \frac{1802.10^4}{90} = 20\,022\ mm^3$$

$= 20{,}022.10^3\ mm^3$

$W_{xo} = 10{,}911.10^3\ mm^3$

U 160 ---- $J_x = 925.10^4\ mm^4$

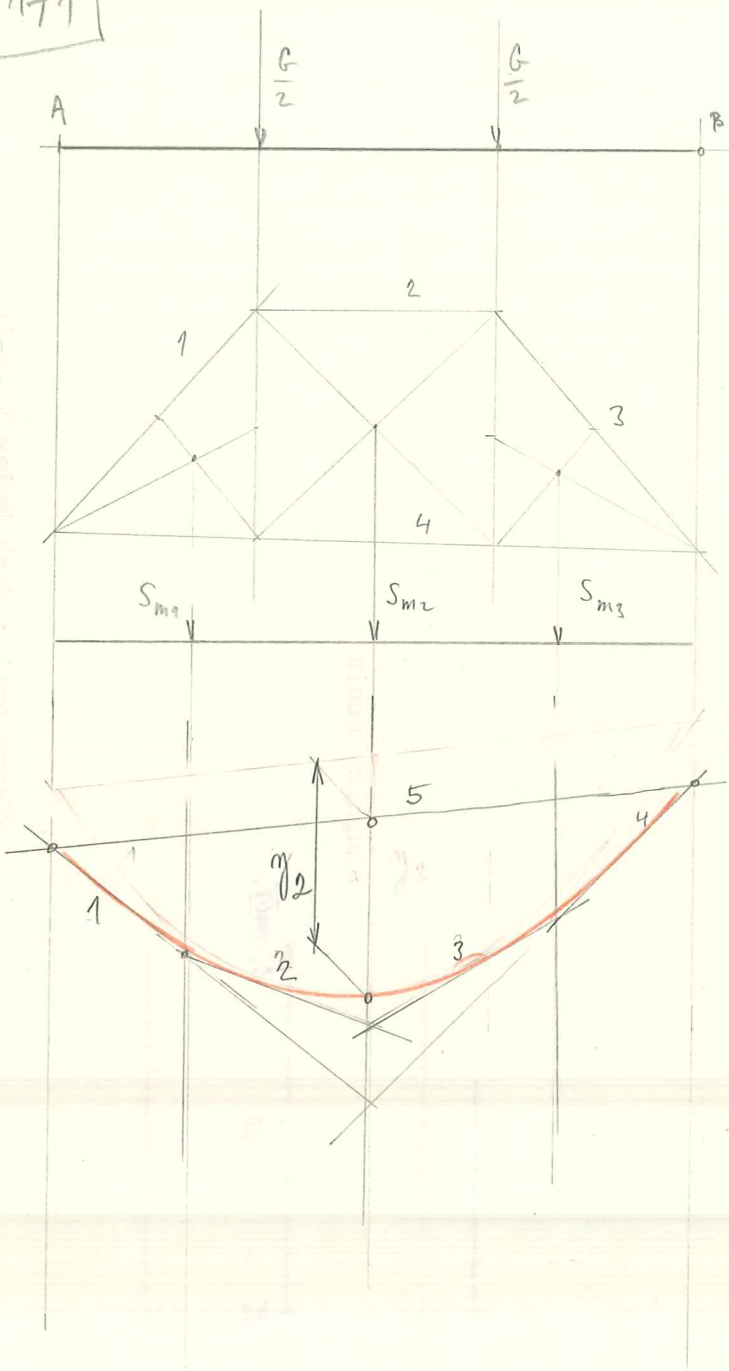
Grafické řešení průhybu

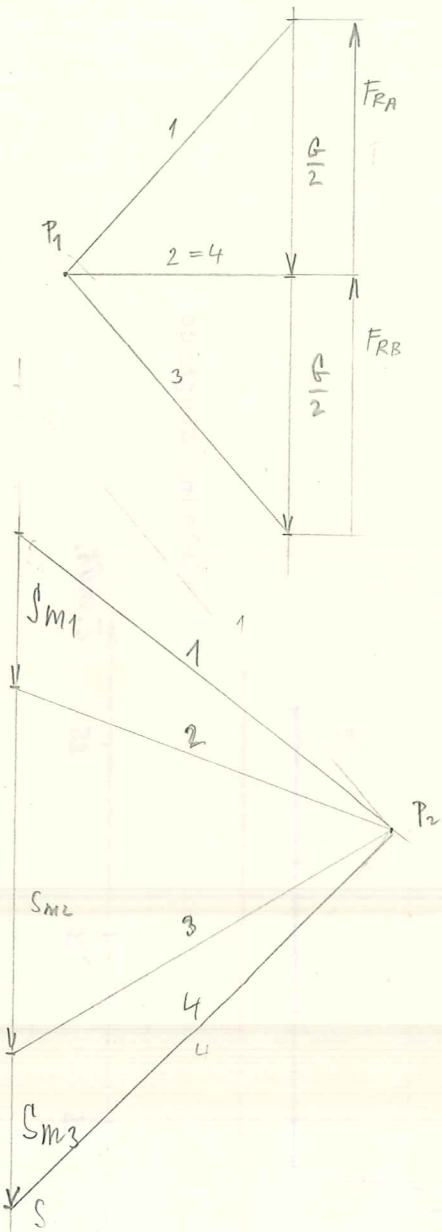
m_L ---- 1 mm $\hat{=}$ 20 mm (85)

m_F ---- 1 mm $\hat{=}$ 1 kN (1000)

$H_1 = 30\ mm$

4.171





$S_{m1} = \frac{1}{2} 26,5 \cdot 30 = 397,5 mm^2$

$S_{m2} = 32 \cdot 30 = 960 mm^2$

$S_{m3} = S_{m1} = 397,5 mm^2$

$1 mm \hat{=} 20 mm^2$

$S_{m1} = 19,87 mm$

$S_{m2} = 48 mm$

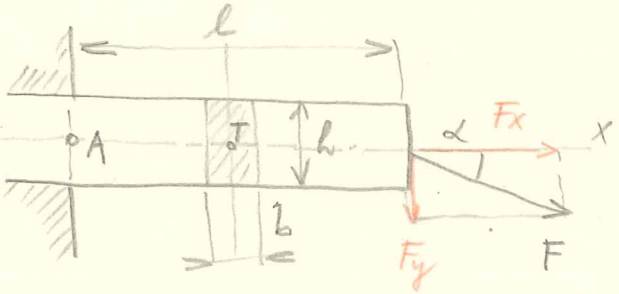
$S_{m3} = 19,87 mm$

$H_2 = 50 mm$

$$\eta_{max} = \eta_2 H_1 H_2 \cdot m_F m_L^3 \cdot m_s \frac{1}{E(2J)} = 23 \cdot 30 \cdot 50 \cdot 1000 \cdot 20^3 \cdot 20 \cdot \frac{1}{2,1 \cdot 10^5 \cdot 925 \cdot 10^4 \cdot 2}$$

$$= 1,42 mm$$

4,239 | A



$b = 120\,mm$
$h = 200\,mm$
$l = 2\,m$
$F = 10\,000\,N$
$\alpha = 30°$

$\sigma_{Dt} = 13\,MPa$

Tah: $F_x = F\cos\alpha = 10\,000 \cdot 0{,}866 = 8{,}66 \cdot 10^3\,N$

$$\sigma_t = \frac{F_x}{S} = \frac{8{,}66 \cdot 10^3}{b \cdot l} = \frac{8{,}66 \cdot 10^3}{120 \cdot 200} = 0{,}36\,MPa$$

Ohyb: $F_y = F\sin\alpha = 10\,000 \cdot 0{,}5 = 5 \cdot 10^3\,N$

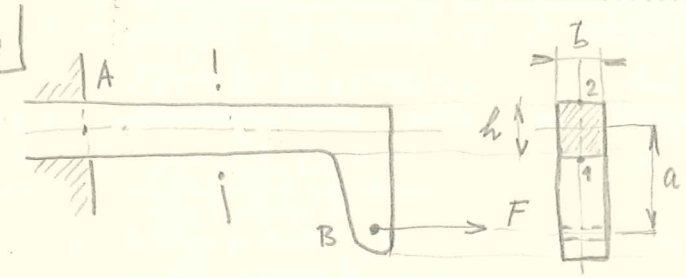
$$\sigma_o = \frac{M_{o\,max}}{W_o} = \frac{F_y \cdot l}{\frac{1}{6} b h^2} = \frac{6 \cdot 5 \cdot 10^3 \cdot 2000}{120 \cdot 200^2} = \frac{60 \cdot 10^6}{120 \cdot 40\,000} = \frac{60 \cdot 10^6}{12 \cdot 4 \cdot 10^5} =$$

$$= 12{,}5\,MPa$$

$$\sigma_{c1} = \sigma_o + \sigma_t = 12{,}5 + 0{,}36 = 12{,}86\,MPa$$

$\sigma_{c1} < \sigma_{Dt}$ – vyhovuje

4.240



$\sigma_1, \sigma_2 = ?$

$F = 1200\,N$ $\quad a = 645\,mm$

$b = 50\,mm$

$h = 30\,mm$

Kontrola rozměrů | $\sigma_{Dt} = \sigma_{Dd} = 115\,MPa$

$M_{ohyb} = F.a = 1200.645 = 774000\,Nmm$

$\sigma_o = \frac{M_{ohyb}}{W_o} = \frac{6.784800}{b\,h^2} = \frac{6.774000}{50.30^2} = \underline{\underline{103,2\,MPa}}$

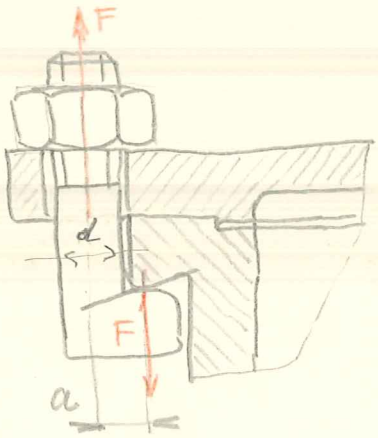
$\sigma_t = \frac{F}{S} = \frac{1200}{b\,h} = \frac{1200}{50.30} = 0,8\,MPa$

$\sigma_{c1} = \sigma_o + \sigma_t = 103,2 + 0,8 = \underline{\underline{104\,MPa}}$

$\sigma_{c1} < \sigma_{Dt}$ .... vyhovuje

4.241

2A



$F = 6000\,N$

M 36

$a = 0,7\,d$

$\sigma_{Dt} = 60\,MPa$

$\sigma_{celk.} = ?$

M36 .... $A_s = 817\,mm^2$ tab.

$d_3 = 31,09 \doteq 31,1\,mm$

$\sigma_t = \frac{F}{A_s} = \frac{6000}{817} = 7,34\,MPa$

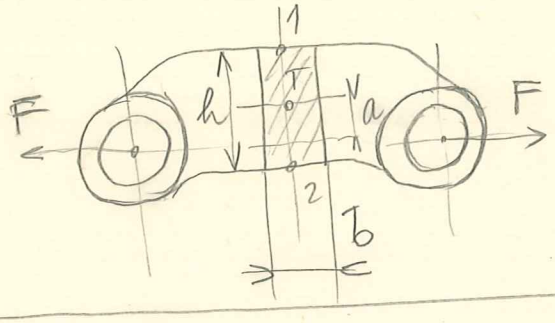
$\sigma_o = \frac{M_{ohyb}}{W_o} = \frac{F.a}{\frac{\pi}{32} d_3^3} = \frac{32\,F.0,7d}{\pi.31,1^3} = \frac{32.6000.0,7.36}{\pi.31,1^3} = 51,2\,MPa$

$\sigma_{celk.} = \sigma_o + \sigma_t = 51,2 + 7,34 = \underline{\underline{58,54\,MPa}}$

$\sigma_{c1} < \sigma_{Dt}$ vyhovuje

(1) 4.242 | Stonbová srážka (A)

4.243 |

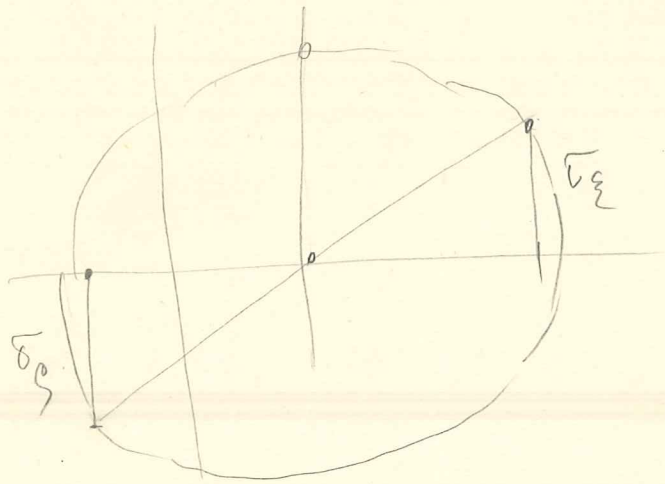


(B)

(2)

$\sigma_\xi =$

$\sigma_\varphi =$



$$\sigma_t = \frac{F}{S} = \frac{2700}{b\,h} = \frac{2700}{10.20} = 13{,}5\ MPa$$

$$M_{o\,max} = F.a = 2700.8 = 21\,600\ N\,mm$$

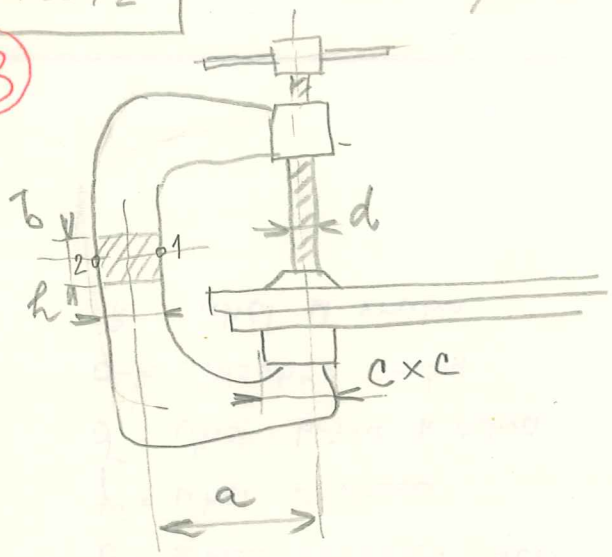
$$\sigma_{o\,max} = \frac{M_{o\,max}}{W_o} = \frac{6.21600}{b\,h^2} = \frac{6.21600}{10.20^2} = 32{,}4\ MPa$$

$$\sigma_1 = -32{,}4 + 13{,}5 = -18{,}9\ MPa$$

$$\sigma_2 = 32{,}4 + 13{,}5 = 45{,}9\ MPa$$

4.242 | Dvě lepené desky

(2B)



$F = ?$
$c = 25\ mm$
$p = 8\ MPa$
$\sigma_1 = ?$
$\sigma_2 = ?$

$b = 20\ mm$
$h = 50\ mm$
$\sigma_{Dt} = 100\ MPa$
$a = 100\ mm$

$$F = p \cdot S = p \cdot c^2 = 8 \cdot 25^2 = 5000\ N$$

$$\sigma_t = \frac{F}{S_1} = \frac{5000}{b \cdot h} = \frac{5000}{20 \cdot 50} = 5\ MPa$$

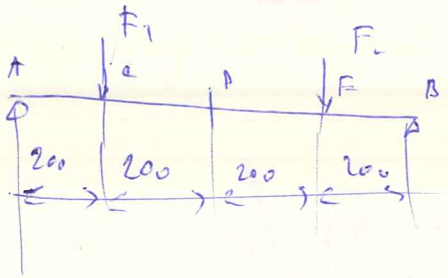
$$M_{o\,max} = F \cdot a = 5000 \cdot 100 = 500\,000\ Nmm$$

$$\sigma_{o\,max} = \frac{M_{o\,max}}{W_o} = \frac{6 \cdot 500\,000}{b\,h^2} = \frac{6 \cdot 500\,000}{20 \cdot 50^2} = 60\ MPa$$

$$\sigma_1 = \sigma_{o\,max} + \sigma_t = 60 + 5 = 65\ MPa \quad \text{vyhovuje}$$

$$\sigma_c < \sigma_{Dt}$$

$$\sigma_2 = -\sigma_{o\,max} + \sigma_t = -60 + 5 = -55\ MPa$$



$F_1 = 200\ N$
$F_2 = 200\ N$

$E = 2,1 \cdot 10^5\ MPa$

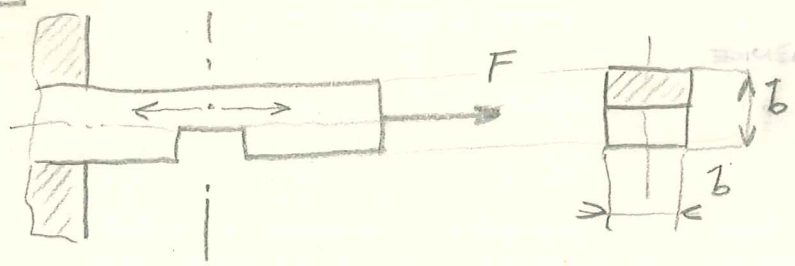
$a = 2$

$\sigma_{Do} = 60\ MPa$

$f_1 = 4,0$

$f_2 = 50$

4. 244



$F = 900\,N$

$\sigma_{Dt} = \sigma_{Dd} = 80\,MPa$

$$\sigma_c \geq \sigma_{Dt}$$

$$\sigma_t = \frac{F}{b\,b\,\frac{1}{2}} = \frac{2F}{b^2}$$

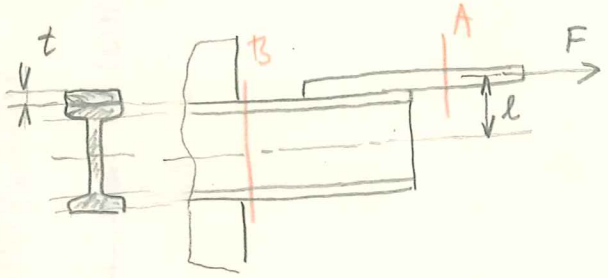
$$\sigma_{0_t} = \frac{M_0}{W_0} = \frac{6F \cdot \frac{b}{4}}{b\left(\frac{b}{2}\right)^2} = \frac{6F}{b^2}$$

$$\sigma_c = \frac{1}{b^2}\left(2F + 6F\right)$$

$$\sigma_{Dt} \cdot b^2 = 8F$$

$$b = \sqrt{\frac{8F}{80}} = \sqrt{\frac{F}{10}} = \sqrt{900} = 30\,mm$$

4.247 | I 120



$t = 10\,mm$

$F = ?$

$\sigma_{Dt} = 120\,MPa$

$\sigma_{Dt} = \frac{F}{S} \Rightarrow F = S \cdot \sigma_{Dt}$

$S = b \cdot t = 58 \cdot 10 = 580\,mm^2$

$F = 580 \cdot 120 = 69{,}6\,kN$

$l = 65$

Tab. $W_x = 54{,}7\,cm^3 = 54{,}7 \cdot 10^3\,mm^3$

Tab. $b = 58\,mm$, $S_I = 1420\,mm^2$

$$\sigma_{Dt} = \frac{F}{S_I} + \frac{M_o}{W_o} = \frac{F}{1420} + \frac{F \cdot l}{54{,}7 \cdot 10^3} = F\left(\frac{1}{1420} + \frac{65}{54{,}7 \cdot 10^3}\right)$$

$$F \frac{54{,}7 \cdot 10^3 + 65 \cdot 1420}{1420 \cdot 54{,}7 \cdot 10^3} = \sigma_{Dt}$$

$$F = \frac{1420 \cdot 54{,}7 \cdot 10^3 \cdot 120}{54{,}7 \cdot 10^3 + 65 \cdot 1420} = \frac{9320{,}88 \cdot 10^6}{147\,080}$$

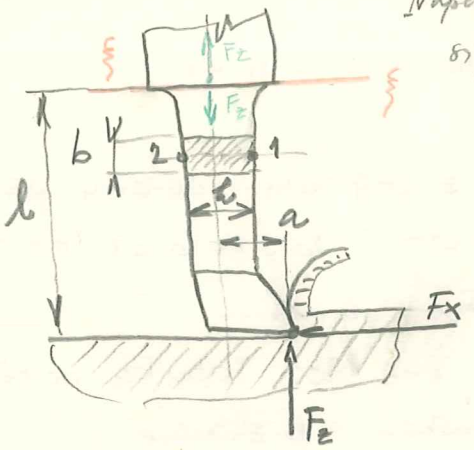
$$= \frac{9320{,}88}{0{,}147} = 63407\,N$$

4,250 | Obrázek míst je zatížen normálovou silami

Napětí τ od posouv. síly zanedbáme

$F_z = F_x = 5120\,N$

$\sigma_{ekv} = ?$ v nebezpečném průřezu

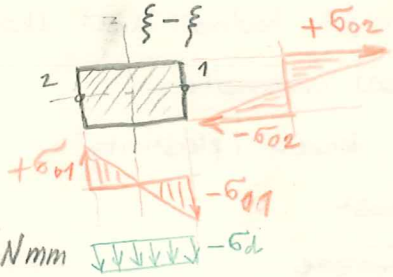


$b = 20\,mm$

$h = 30\,mm$

$a = 20\,mm$

$l = 120\,mm$



$M_{o1} = F_z \cdot a$

$M_{o2} = F_x \cdot l$

$M_{o1} = 5120 \cdot 20 = 1{,}024 \cdot 10^5\,Nmm$

$\sigma_{o1} = \frac{M_{o1}}{W_o} = \frac{6 \cdot 1{,}024 \cdot 10^5}{b h^2} = \frac{6 \cdot 1{,}024 \cdot 10^5}{20 \cdot 30^2} = 34{,}13\,MPa$

$\sigma_d = \frac{F_z}{bh} = \frac{5120}{20 \cdot 30} = 8{,}53\,MPa$

$M_{o2} = 5120 \cdot 120 = 6{,}144 \cdot 10^5\,Nmm$

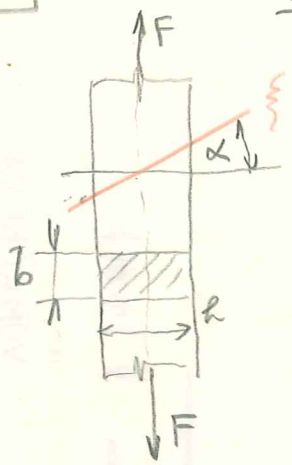
$\sigma_{o2} = \frac{6 M_{o2}}{b h^2} = \frac{6 \cdot 6{,}144 \cdot 10^5}{20 \cdot 30^2} = 204{,}8\,MPa$

Vlákno 1: $\sigma_{ekv} = \sigma_{o2} - \sigma_{o1} - \sigma_d = 204{,}8 - 34{,}13 - 8{,}53 = 162{,}14\,MPa$

2: $\sigma_{ekv} = \sigma_{o1} - \sigma_{o2} = 34{,}13 - 204{,}8 - 8{,}53 = -179{,}2\,MPa$

4.255

Normálová a tečná namáhání



$F = 30\,000\ N$

$b = 15\ mm$

$h = 30\ mm$

Počátek v grafu ξ (M, Z mají)

ξ s $\alpha = 30°$

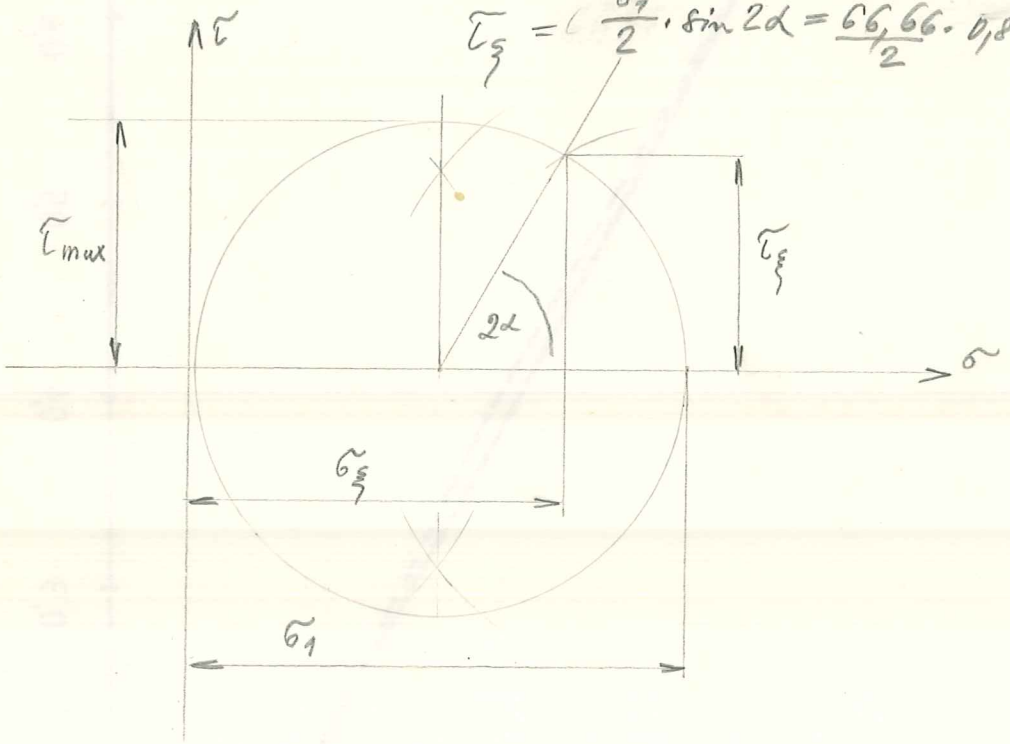
~~$\sigma_\xi = \sigma_1 \cos^2\alpha = \frac{F}{S}\cos^2 30 = \frac{30\,000}{15 \cdot 30} \cdot 0{,}866^2 = 57{,}73\ MPa$~~

~~$\tau_\xi = \sigma_1 \sin 2\alpha = \frac{F}{S}\sin 60 = 57{,}73\ MPa$~~

$$\sigma_1 = \frac{F}{S} = \frac{30\,000}{15 \cdot 30} = 66{,}66\ MPa$$

$$\sigma_\xi = \sigma_1 \cos^2\alpha = 66{,}66 \cdot \cos^2\alpha = 66{,}66 \cdot 0{,}866^2 = 49{,}92\ MPa$$

$$\tau_\xi = \frac{\sigma_1}{2} \cdot \sin 2\alpha = \frac{66{,}66}{2} \cdot 0{,}866 = 28{,}86\ MPa$$



$\sigma_\xi = 50\ MPa$

$\tau_\xi = 28{,}5\ MPa$

$\tau_{max} = 33\ MPa$

4.262 Hřídel dutý $d_1 = 100$ mm, $d_2 = 70$ mm — Dílec/trubka

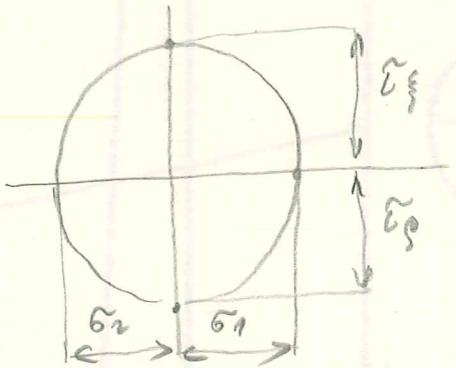
obr. 140 $M_k = 6000$ Nm (Nakreslete M. kroutící)

Vypočtěte σ_1, σ_2

$$W_k = \frac{\frac{\pi}{32}(d_1^4 - d_2^4)}{\frac{d_1}{2}} =$$

$$= \frac{\frac{\pi}{32}(100^4 - 70^4)}{50} = \frac{\frac{\pi}{32} \cdot 7599000}{50} = 149201{,}6 \text{ mm}^3$$

$$\tau_k = \frac{M_k}{W_k} = \frac{6000000}{149201{,}6} = 40{,}21 \text{ MPa}$$



$\sigma_1 = 40{,}21$

$\sigma_2 = -40{,}21$

4.263 $M_k = 275$ Nm — hřídel je namáhán pouze kroutícím momentem

$\sigma_{1,2} = \pm 30$ MPa

$\tau = \sigma_1 = 30$ MPa

$$W_k = \frac{M_k}{\tau_k} = \frac{275000}{30} = 9166{,}6 \text{ mm}^3$$

$$W_k = \frac{\pi}{16} d^3 \Rightarrow d = \sqrt[3]{\frac{16 W_k}{\pi}} = \sqrt[3]{\frac{16 \cdot 9166{,}6}{\pi}} = 36 \text{ mm}$$

$F_{II} = ?$
$F_{RÁZ.} = ?$

$d = 12\,mm$
$l = 1\,m$

11 375
$k = 1,6$

$\frac{0,7.370}{k} = 161,9\,MPa$

$C_{II} = 0,85$ (St.

$C_{RÁZ} = 0,65$

$= 0,85.164 = \underline{137,4\,MPa}$

$= \underline{90,6\,MPa}$

$= \frac{\pi}{32}.12^3 = 169,6\,mm^3$

$\dot{}.W_0 = 90,61.169,6 = 15367,4\,Nmm$

$= \frac{15367,4}{1000} = \underline{15,36\,N}$

$F = 17\,N$ $\quad y = 0,25\,mm$

$b = 22\,mm$ $\quad$ Cu-fosfor. bronz $E = 1,1.10^5\,MPa$

$l = 34\,mm$ $\quad \sigma_{DO} = 60\,MPa$

$h = ?$

$$y = \frac{Fl^3}{3EJ} \Rightarrow J = \frac{Fl^3}{3yE}$$

$$J = \frac{17.34^3}{3.0,25.1,1.10^5} = \underline{8,099\,mm^4}$$

$$J = \frac{1}{12}bh^3$$

$$h = \sqrt[3]{\frac{12J}{b}} = \sqrt[3]{\frac{12.8,1}{22}} = 1,64\,mm$$

$$\sigma_o = \frac{M_{ohyb}}{W_o} = \frac{Fl}{\frac{1}{6}bh^2} = \frac{6.17.34}{22.1,64^2} = \underline{58,6\,MPa}$$

VZPĚR

4.298 $d_1 = 40$ mm $d_2 = 34$ mm $l = 2$ m konce kloubově uložený $E = 2{,}1 \cdot 10^5$ MPa

$F_{kr} = ?$ podle Eulera

$$F_{kr} = \frac{\pi^2 E J}{l_{red}^2} = \frac{\pi^2 \cdot 2 \cdot 10^5 \cdot 60064{,}69}{2000^2} =$$

$$J = \frac{\pi}{64}(40^4 - 34^4) = 60064{,}69 \text{ mm}^4$$

$$= 29640 \text{ N}$$

4.299 Zadání 4.298 konce přivařený (vetknutý) $l_{red} = \frac{l}{2}$

$$F_{kr} = 4 \cdot F_{kr\ kloub} = 4 \cdot 29640 = 118560 \text{ N}$$

4.300 Zadání 4.298 1 konec přivařený druhý kloubový $l_{red} = \frac{l}{\sqrt{2}}$

$$F_{kr} = 2 F_{kr\ kloub} = 2 \cdot 29640 = 59280 \text{ N}$$

4.301 PÍSEMKA $j' = ?$ pro kruhový průřez

$$j' = \sqrt{\frac{J}{S}} = \sqrt{\frac{\frac{\pi}{64} d^4}{\frac{\pi}{4}}} = d\sqrt{\frac{1}{16}}$$

$$J = \frac{\pi}{64} d^4 \qquad S = \frac{\pi d^2}{4}$$

$$= \frac{d}{4}$$

4.302 PÍSEMKA $j'_{min} = ?$ pro  $h : b = 2$

$$J_{min} = \frac{1}{12} b^3 h = \frac{1}{12} b^3 2b = \frac{b^4}{6}$$

$h = 2b$

$$j'_{min} = \sqrt{\frac{J_{min}}{S}} = \sqrt{\frac{\frac{b^4}{6}}{2b^2}} = \frac{b}{\sqrt{12}}$$

$$S = b \cdot h = 2b^2$$

4.303 $\lambda = ?$ oba konce kloubově uložené $d = 20$ mm, $l = 1200$ mm

$$j' = \frac{d}{4} = \frac{20}{4} = 5 \qquad l_{red} = l$$

4.304 $\lambda = ?$ I 80 ČSN 42 0076 konce oba vetknuté

$l = 4000\ mm$

$J_{min} = J_y = 6{,}29\ cm^4 =$ ST str. 403

$= 6{,}29 \cdot 10^4\ mm^4$

$S = 758\ mm^2$

$l_{red} = \frac{l}{2} = \frac{4000}{2} = 2000\ mm$

$j_{min} = \sqrt{\frac{J_{min}}{S}} = \sqrt{\frac{6{,}29 \cdot 10^4}{758}} = 9{,}109$

$\lambda = \frac{l_{red}}{j_{min}} = \frac{2000}{9{,}109} = \underline{219{,}56}$

4.305 $\lambda_m = 105$, d $l_{red} = ?$

$j = \frac{d}{4}$

$\lambda_m = \frac{l_{red}}{j}$

$l_{red} = \lambda_m \cdot j = 105 \frac{d}{4} = \underline{\underline{26{,}25\,d}}$

4.306 Zjistěte $\lambda_m = ?$ $\sigma_u = 200\ MPa$, $E = 2{,}0 \cdot 10^5\ MPa$

$\lambda_m = \pi \sqrt{\frac{E}{\sigma_u}} = \pi \sqrt{\frac{2 \cdot 10^5}{200}} = \pi \sqrt{1000} = \underline{\underline{99{,}34}}$

4.307 $\lambda_m = ?$ pro dřevo $\sigma_u = 20\ MPa$, $E = 1{,}2 \cdot 10^4\ MPa$

$\lambda_m = \pi \sqrt{\frac{E}{\sigma_u}} = \pi \sqrt{\frac{1{,}2 \cdot 10^4}{20}} = \underline{\underline{76{,}95}}$

4,308 Zda platí Euler. vzor.

$d = 100\,mm$ — tloušťka válcování 11453

$l = 3\,m$ 937

$j' = \frac{d}{4} = \frac{100}{4} = 25\,mm$

$l_{red} = l$

$\lambda = \frac{l_{red}}{j'} = \frac{3000}{25} = \underline{\underline{120}}$ platí

4,309 $d_1 = 160\,mm$ $\lambda = 47$ jako λ má být pro plná ocelová kruhová průřez

$d_2 = 50\,mm$

$S = \frac{\pi}{4}(d_1^2 - d_2^2) = \frac{\pi}{4}(160^2 - 50^2) = 18142{,}69\,mm^2$

$d = \sqrt{\frac{4S}{\pi}} = 151{,}98\,mm$

$\lambda = \frac{l_{red}}{j'} \Rightarrow l_{red} = \lambda \cdot j' = \lambda \sqrt{\frac{J_0}{S}}$ $\lambda_1 = \frac{l}{j'} = \frac{l}{\sqrt{\frac{J}{S}}}$

$\lambda_1 = \frac{\lambda \sqrt{\frac{J_0}{S}}}{\sqrt{\frac{J}{S}}} = \lambda \sqrt{\frac{\frac{J_0}{S}}{\frac{J}{S}}} = \lambda \sqrt{\frac{\frac{\pi}{64}(d_1^4 - d_1^4)}{\frac{\pi}{64} d^4}} =$

$= \lambda \sqrt{\frac{100^4 - 50^4}{151{,}98^4}} = 47 \sqrt{\frac{93750000}{23097{,}92}} = 47 \cdot 63 = 2994{,}3$

$\lambda = 47$ $\lambda = \frac{l}{j'} = \frac{l}{\sqrt{\frac{J}{S}}} \Rightarrow l = \lambda \sqrt{\frac{J}{S}}$

$\lambda_1 = \frac{l}{\sqrt{\frac{J_0}{S}}} = \frac{\lambda \sqrt{\frac{J}{S}}}{\sqrt{\frac{J_0}{S}}} = \lambda \sqrt{\frac{J}{J_0}} = \lambda \sqrt{\frac{\frac{\pi}{64}(d_1^4 - d_2^4)}{\frac{\pi}{64} d^4}}$

$= \lambda \sqrt{\frac{160^4 - 50^4}{151{,}98^4}} = \lambda \sqrt{\frac{93750000}{53513926}} = \lambda \sqrt{\frac{649109999}{53513926}}$

$= \lambda \sqrt{1{,}216669} = \lambda\, 1{,}103027 = 47 \cdot 1{,}103027 =$

$\underline{\underline{51{,}84}}$

$\frac{6{,}5}{\sqrt{7}} \doteq 52$

4.310 PÍSEMKA

$d_1 = 42\ mm$; $l = 2\ m$ | $F = 8000\ N$

$d_2 = 36\ mm$

konec kloub. uložení

Ukazovat a kontrola ☆

$E = 2{,}0 \cdot 10^5\ MPa$

$\sigma_u = 200\ MPa$

$k = 4$

$$\lambda = \frac{l_{red}}{j} = \frac{2000}{13{,}83} = 144{,}6 \qquad \lambda_m = \pi\sqrt{\frac{E}{\sigma_u}} = \pi\sqrt{\frac{2{,}0 \cdot 10^5}{200}} = 99{,}3$$

$$j = \sqrt{\frac{\frac{\pi}{64}(d_1^4 - d_2^4)}{\frac{\pi}{4}(d_1^2 - d_2^2)}} = \sqrt{\frac{1432080}{16 \cdot 468}} = \sqrt{191{,}24} = 13{,}829$$

$$F_{kr} = \frac{\pi^2 E J}{l_{red}^2} = \frac{\pi^2 \cdot 2 \cdot 10^5 \cdot 70297{,}1}{2000^2} = 34690{,}23\ N$$

$$J = \frac{\pi}{64}(1432080 = 70297{,}1$$

$$k = \frac{F_{kr}}{F} = \frac{34690{,}23}{8000} = 4{,}33$$ vyhovuje

---

4.311 PÍSEMKA obr. 159

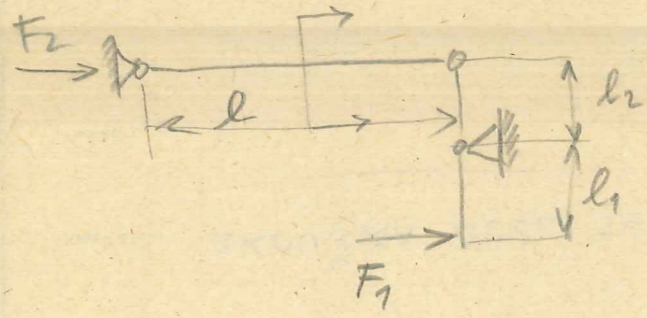
$F_1 = 4000\ N$

$E = 2 \cdot 10^5\ MPa$

$l_1 = 50\ mm$; $b = 5\ mm$; $l = 200\ mm$

$l_2 = 60\ mm$ $h = 20\ mm$ $k = 3$

11500



$$F_1 l_1 = F_2 l_2$$

$$F_2 = \frac{F_1 l_1}{l_2} = \frac{4000 \cdot 50}{60} = 3333{,}3\ N$$

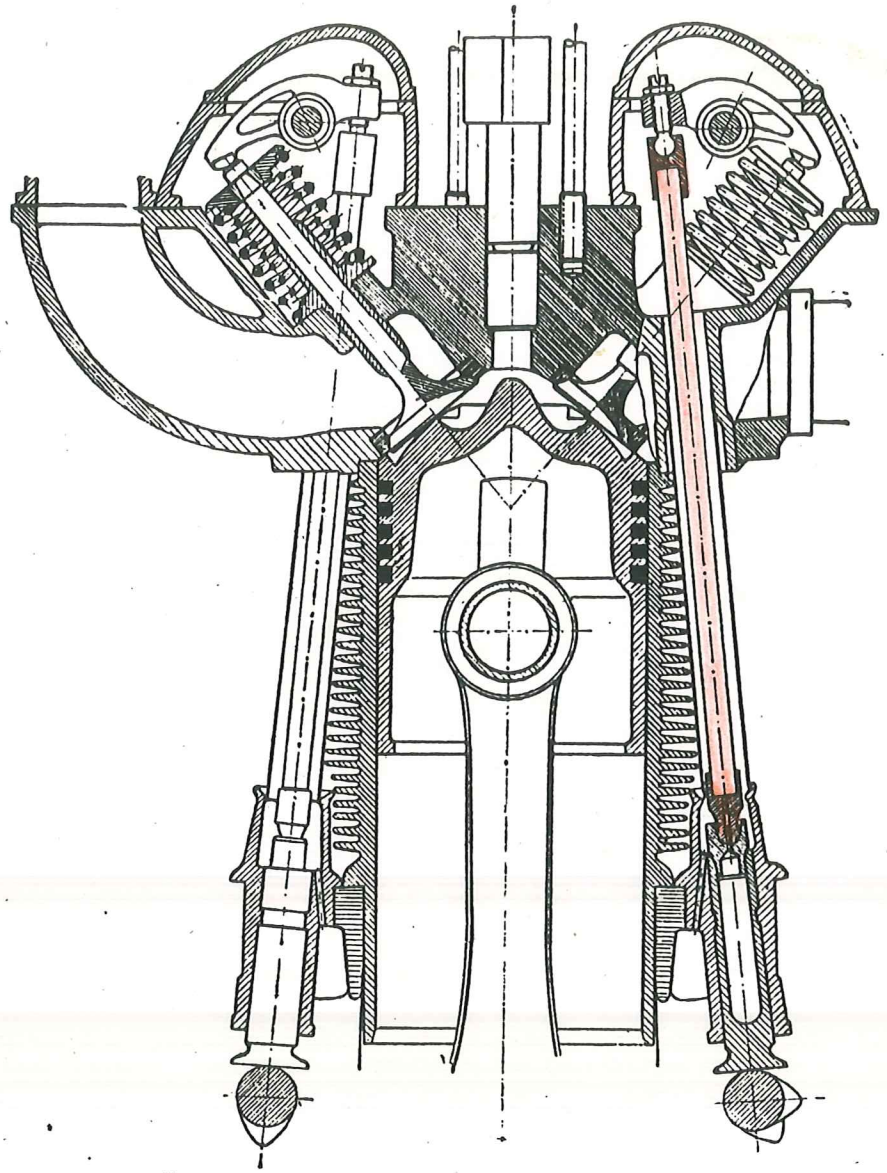
$$\lambda = \frac{l_{red}}{j'_{min}} = \frac{200}{1{,}443} = 138 \qquad j'_{min} = \sqrt{\frac{J}{S}} = \sqrt{\frac{\frac{1}{12} b^3 h}{b \cdot h}} = \frac{b}{\sqrt{12}} = 1{,}443$$

Euler.

$$F_{kr} = \frac{\pi^2 E J_{min}}{l_r^2} = \frac{\pi^2 \cdot 2 \cdot 10^5 \cdot 208{,}3}{200^2} = 10279\ N$$

$$J_{min} = \frac{1}{12} b^3 h = \frac{1}{12} \cdot 5^3 \cdot 20 = 208{,}3$$

torů klikové ústrojí (mechanismus), které je často vyosované. Rozvodové ústrojí je nejčastěji ventilové, řízené vačkovým ústrojím. Obojí ústrojí je za běhu stroje zatěžováno proměnlivě, což se projevuje chvěním, které někdy může dokonce porušit kmitající strojní součást. Proto na dlouhých hřídelích jsou tlumiče zkrutových (torsiálních) vibrací. Pru-



Obr. 8-XI. Řez motorem s přímým vstřikem, Tatra 111.

žiny ventilů mají buď tlumiče chvění, nebo nestejné stoupání závitů. Nerovnoměrnost točivého momentu motoru je vyrovnávána větším počtem válců a setrvačníkem. U několikaválcových motorů jsou odstraněny také vlivy setrvačných sil. Písty se zhotovují z lehkých slitin, zejména hliníkových, aby setrvačné síly byly malé i při velkém počtu otáček. Aby se nepoškodily motory s těžkým klikovým ústrojím, jsou jejich otáčky omezovány regulátory.

Rozvodové ústrojí je poháněno ozubeným nebo řetězovým soukolím od klikového hřídele. Vačkový hřídel musí být při zmontování motoru nastaven tak, aby časování rozvodu vyhovovalo příslušným polohám

4.312 $d = ?$ $E = 2{,}1 \cdot 10^5$ MPa

$F = 78\,000$ N kl. nebezp. obou konců

$l = 1350$ mm

$k = 3{,}5$

$F_{kr} = k \cdot F = 3{,}5 \cdot 78000 = \underline{273\,000\text{ N}}$

$$F_{kr} = \frac{\pi^2 E J_{min}}{l_{red}^2}$$

$$J_{min} = \frac{F_{kr} l_{red}^2}{\pi^2 E} = \frac{273000 \cdot 1350^2}{\pi^2 \cdot 2 \cdot 10^5} = 252057{,}9 \text{ mm}^4$$

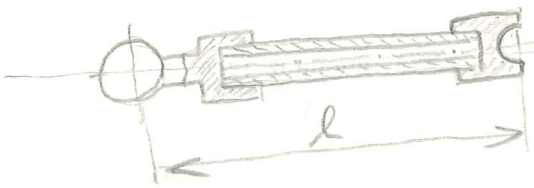
$$J_{min} = \frac{\pi}{64} d^4 \Rightarrow d = \sqrt[4]{\frac{64 J_{min}}{\pi}} = \sqrt[4]{\frac{64 \cdot 252057{,}9}{\pi}} = 47{,}6 \text{ mm}$$

$\underline{d = 48 \text{ mm}}$

kontrola $\lambda = \frac{l_{red}}{j'} = \frac{1350}{12} = \underline{112{,}5}$

$$j' = \sqrt{\frac{J}{S}} = \frac{d}{4} = \frac{48}{4} = 12$$

4.313



$D_1 = 6$ mm

$D_2 = 4{,}8$ mm

$l = 288$ mm

$F = 226$ N

$E = 2{,}15 \cdot 10^5$ MPa

$k = 4$

$$J = \frac{\pi}{64}\left(D_1^4 - D_2^4\right) = \frac{\pi}{64}\left(6^4 - 4{,}8^4\right) = 37{,}559 \text{ mm}^4$$

$$S = \frac{\pi}{4}\left(D_1^2 - D_2^2\right) = \frac{\pi}{4}\left(6^2 - 4{,}8^2\right) = 10{,}178 \text{ mm}^2$$

$$j' = \sqrt{\frac{J}{S}} = \sqrt{\frac{37{,}56}{10{,}2}} = 1{,}918$$

$$\lambda = \frac{l_{red}}{j'} = \frac{288}{1{,}918} = 150$$

Euler. $F_{kr} = \frac{\pi^2 E J}{l_{red}^2} = \frac{\pi^2 \cdot 2{,}15 \cdot 10^5 \cdot 37{,}56}{288^2} = 960{,}9 \text{ N}$

$F_{kr} = 960{,}9 \text{ N} > 226 \text{ N}$

4, 314 | $D_1 = 90\,mm$, $D_2 = 74\,mm$ dut/plná tyč
$F = 8000\,N$, $l = 6\,m$, $E = 2{,}16 \cdot 10^5\,MPa$
obr. $k = 12$ Kontrola rovnováhy

$$J = \frac{\pi}{64}\left(D_1^4 - D_2^4\right) = \frac{\pi}{64}\left(90^4 - 74^4\right) = 1748660{,}7\,mm^4$$

$$S = \frac{\pi}{4}\left(D_1^2 - D_2^2\right) = \frac{\pi}{4}\left(90^2 - 74^2\right) = 2060{,}8\,mm^2$$

$$j' = \sqrt{\frac{J}{S}} = \sqrt{\frac{1748660{,}7}{2060{,}8}} = 29{,}129$$

$$\lambda = \frac{l_{red}}{j'} = \frac{6000}{29{,}129} = 205{,}98 \qquad l_{red} = l$$

Euler $\lambda > \lambda_m$

$$F_{kr} = \frac{\pi^2 E J}{l_{red}^2} = \frac{\pi^2 \cdot 2{,}16 \cdot 10^5 \cdot 1748660{,}7}{6000^2} = 103551{,}5\,N$$

$$F = \frac{F_{kr}}{k} = \frac{103551{,}5}{12} = \underline{\underline{8629{,}3\,N}} \quad \text{ohrožuje}$$

3, 315 | $l = 1\,m$ $b = 20\,mm$, $\lambda = 100$
$h = 40\,mm$ | $E = 2{,}1 \cdot 10^5\,MPa$
Tělesové ohýbání

$$\sigma_{kr} = 335 - 0{,}62\lambda \;,\; k = 10$$

$F_{kr} = ?$ $F = ?$

$$J_{min} = \frac{1}{12} b^3 h = \frac{1}{12} 20^3 \cdot 40 = 26666{,}6\,mm^4$$

$$j'_{min} = \sqrt{\frac{\frac{1}{12} b^3 h}{bh}} = \frac{b}{\sqrt{12}} = \frac{20}{3{,}464} = 5{,}773$$

$$\lambda = \frac{l}{j'_{min}} = \frac{1000}{5{,}773} = 173{,}2$$

Výpočet podle Eulera

$$F_{kr} = \frac{\pi^2 E J}{l_{red}^2} = \frac{\pi^2 \cdot 2{,}1 \cdot 10^5 \cdot 26666{,}6}{1000^2} = 55269{,}6\,N$$

$$F = \frac{F_{kr}}{k} = \underline{\underline{5526{,}9\,N}}$$

4.316 | $l = 2000\ mm$, $d = 250\ mm$ ze dřeva, kloubové uložení, $F = 100\ kN$, $\lambda_m = 100$

$k = ?$

$J = \frac{\pi}{64} d^4 = \frac{\pi}{64} 250^4 =$

$j' = \frac{d}{4} = \frac{250}{4} = 62{,}5\ mm$

$\lambda = \frac{l_{red}}{j'} = \frac{2000}{62{,}5} = 32$ vypočítat podle TETMAJERA

$\sigma_{kr} = 29{,}3 - 0{,}194\lambda = 29{,}3 - 0{,}194 \cdot 32 = \underline{23{,}092\ MPa}$

$F_{kr} = \sigma_{kr} \cdot S = 23{,}1 \cdot \frac{\pi}{4} 250^2 = 1133918{,}5\ N$

$k = \frac{F_{kr}}{F} = \frac{1133918}{100\,000} = \underline{11{,}34}$

4.317 | Velkmuté konce ZADÁNÍ NENÍ DOBRÉ!

$l = 2000\ mm$, $F = 30\ kN$

$d_1 = 40\ mm$, $d_2 = ?$, $k = 2{,}5$

$F_{kr} = k \cdot F = 2{,}5 \cdot 30000 = 75\,000\ N$

$l_{red} = \frac{l}{2}$

$\lambda = \frac{l_{red}}{j'} = \frac{1000}{10} = 100 \quad \left| \quad j = \frac{d_1}{4} = 10 \right.$

$d = 34$

$\lambda_\odot < \lambda_m$

Tetmajer $\lambda = 50$ – vdlhodrevo

$\sigma_{kr} = 289 - 0{,}82\lambda \doteq 248\ MPa$

$F_{kr} = \sigma_{kr} \cdot S$

$j' = \sqrt{\frac{J}{S}}$

$S = \frac{F_{kr}}{\sigma_{kr}} = \frac{75\,000}{248} = 302{,}4\ mm^2$

$S = \frac{\pi}{4} d_1^2 - \frac{\pi}{4} d_2^2$

$-S + \frac{\pi}{4} d_1^2 = + \frac{\pi}{4} d_2^2$

$\lambda_{skut} = \frac{l_{red}}{j'} = \frac{1000}{13,25} = 75,47$

$J = \frac{\pi}{64}(40^4 - 34,8^4) = 53669,6\ mm^4$

$S = \frac{\pi}{4}(40^2 - 34,8^2) = 305,47\ mm^2$

$j' = \sqrt{\frac{J}{S}} = \sqrt{\frac{53669,6}{305,97}} = 13,25\ mm$

$\sigma_{kr} = 289 - 0,82 \cdot 75,47 = 227,11\ MPa$

$F_{kr} = S \cdot \sigma_{kr} = 227,11 \cdot 305,47 = 69375,12\ N$

$k = \frac{F_{kr}}{F} = \frac{69375,12}{30000} = \underline{2,31}$ nevyhovuje

zvolím $\underline{d_2 = 34\ mm}$ (asi vyhovuje – přepočítat)

17,75

4.318 $F = 300 \cdot 10^3\ N$ $l = 1500\ mm$ 11500 --- $E = 2 \cdot 10^5\ MPa$

$d = ?$ $k = 8$

Obr. 4.163

Kloubové uložení

$l_{red} = l$

Euler $F_{kr} = \frac{\pi^2 E J_{min}}{l_{red}^2}$ $F_{kr} = 8 \cdot 300 \cdot 10^3 = 24 \cdot 10^5\ N$

$J = \frac{F_{kr} l_{red}^2}{\pi^2 E} = \frac{24 \cdot 10^5 \cdot 1500^2}{\pi^2 \cdot 2 \cdot 10^5} = \underline{273583\ mm^4}$

$J = \frac{\pi}{64} d^4$

$d = \sqrt[4]{\frac{64 \cdot J}{\pi}} = \sqrt[4]{\frac{64 \cdot 273583}{\pi}} = 86,4\ mm$

$S = 5862,9\ mm^2$ $j' = \frac{d}{4} = 21,6$

$\lambda = \frac{1500}{21,6} = \underline{69,4}$

TETMAJER

$\sigma_{kr} = 335 - 0,62\lambda = 335 - 0,62 \cdot 69,4 = 291,9\ MPa$

$F_{kr} = 8,0 \cdot 300 \cdot 10^3 = 24 \cdot 10^5$

$S = \frac{F_{kr}}{\sigma_{kr}} = \frac{24 \cdot 10^5}{291,9} = 8221,9\ mm^2$

$d = \sqrt{\frac{4S}{\pi}} = \sqrt{\frac{4 \cdot 8221,9}{\pi}} = 102\ mm$

$\lambda = \frac{l_{red}}{j} = \frac{1500}{26,35} = 56,9$ $\quad j = \frac{d}{4} = 26,35$

$\sigma_{kr} = 335 - 0,62 \cdot 56,9 = 299,7\ MPa$

$S = 8724,8\ mm^2$

$F_{kr} = S \cdot \sigma_{kr} = 299,7 \cdot 8724,8 = 26,148 \cdot 10^5\ N$

$k = \frac{F_{kr}}{F} = 8,7$ Vyhovuje

4.319 Sloup z litiny 42 2424 $F = 500 \cdot 10^3\ N$

ob. 4.164 $D = 250\ mm$, $d = 230\ mm$

$l = 3\ m$, $F = 500 \cdot 10^3\ N$

$E = 1,08 \cdot 10^5\ MPa$

$k = 2$

$J = \frac{\pi}{64}(D^4 - d^4) = \frac{\pi}{64}(250^4 - 230^4) = 54379365\ mm^4$

$S = \frac{\pi}{4}(D^2 - d^2) = \frac{\pi}{4}(250^2 - 230^2) = 7539,6\ mm^2$

$j = \sqrt{\frac{J}{S}} = \sqrt{\frac{54379365}{7539,6}} = \sqrt{7212,49} = 84,926\ mm$

$\lambda = \frac{l_{red}}{j} = \frac{3000}{84,926} = 35,32$

TETMAJER

$\sigma_{kr} = 776 - 1,2\lambda + 0,053\lambda^2 = 776 - 1,2 \cdot 35,32 + 0,053 \cdot 35,32^2 =$

$= 776 - 42,38 + 66,11 = 799,73\ MPa$ ~~418,3 MPa~~

$F_{kr} = S \cdot \sigma_{kr} = 7539,6 \cdot 418,3 = 3153890\ N$

$k_e = \frac{F_{kr}}{F} = \frac{2156690}{500 \cdot 10^3} = 4,3$ ~~2156690 N~~

4.320 | Ojnica

Obr. 4.165

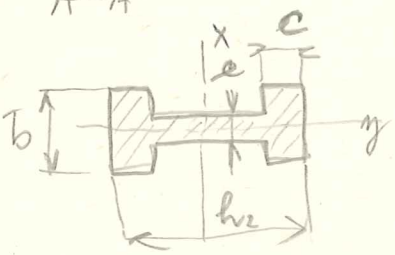
$F = 25\,830\ N$ / $l = 255\ mm$

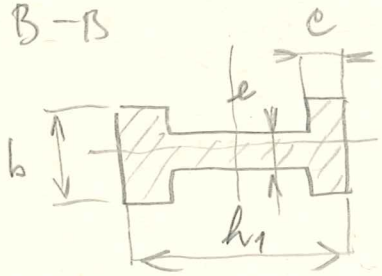
A-A — $h_2 = 42\ mm$

$b = 25\ mm$

$c = e = 8{,}5\ mm$

B-B — $h_2 = 38\ mm$





$k = 7$ — 42 26 50

$E = 2{,}2 \cdot 10^5\ MPa$

$\sigma_{kt} = 250\ MPa$

AA

$J_x = \frac{1}{12} b h_2^3 - \frac{1}{12}(b-e)(h_2-2c)^3$ | $l_{red} = l$

$= \frac{1}{12} 25 \cdot 42^3 - \frac{1}{12} 16{,}5 \cdot 25^3 =$

$= 154350 - 21484{,}37 = 132865{,}6\ mm^4$

$S = b h_2 - [(b-e)\cdot(h_2-2c)] = 25 \cdot 42 - 16{,}5 \cdot 25 = 637{,}5\ mm^2$

$j_x = \sqrt{\frac{J_x}{S}} = \sqrt{\frac{132865{,}6}{637{,}5}} = 14{,}43$

$\lambda = \frac{l_{red}}{j_x} = \frac{255}{14{,}43} = 17{,}6$

Kontrola na tlak

$\sigma_d = \frac{F}{S} = \frac{25\,830}{637{,}5} = 40{,}51\ MPa$

$\sigma_{Dd} = \frac{\sigma_{Pt}}{k} \cdot c_{II} = \frac{500}{7} \cdot 0{,}85 = 60\ MPa$

$\sigma_d < \sigma_{Dd}$

$J_y = \frac{1}{12} b^3 2c + \frac{1}{12} e^3 (h_2 - 2c) =$ | $l_{red} = \frac{l}{2}$

$= \frac{1}{12} 25^3 \cdot 17 + \frac{1}{12} 8{,}5^3 \cdot 25 = 22135{,}4 - 1279{,}4 = 20855{,}9\ mm^4$

$j_y = \sqrt{\frac{20855{,}9}{637{,}5}} = 32{,}71$

$\lambda = \frac{l_{red}}{j_y} = \frac{255}{2 \cdot 32{,}71} = 3{,}89$

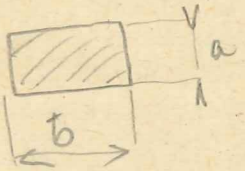
4.321 NE

4.322 NE

4.323

PÍSEMKA Razmik

$l = 100\,mm$; $F = 70\,100\,N$; $E = 2{,}1 \cdot 10^5\,MPa$; $\sigma_{Dd} = 280\,MPa$; $a = 30\,mm$; $b = 40\,mm$



$$l_{red} = \frac{l}{2}$$

$$J_{min} = \frac{1}{12} b a^3 = \frac{1}{12} 40 \cdot 30^3 = 90\,000\,mm^4$$

$$S = b \cdot a = 30 \cdot 40 = 1200\,mm^2$$

$$j = \sqrt{\frac{J}{S}} = \sqrt{\frac{90\,000}{1200}} = \sqrt{75} = 8{,}66$$

$$\lambda = \frac{l_{red}}{j} = \frac{100}{2{,}8{,}66} = 5{,}773$$

Kontrola na tlak

$$\sigma_d = \frac{F}{S} = \frac{70\,100}{1200} = 58{,}41\,MPa$$

$\sigma_d < \sigma_{Dd}$ vyhovuje

4.324

PÍSEMKA

$t = 1{,}5\,mm$ 42 4201

$d = 3\,mm$ $\tau_{PS} = 100\,MPa$

$l_1 = 60\,mm$

$E = 2{,}15 \cdot 10^5\,MPa$; $k = 2$ až 3

$$F = \pi d t \cdot \tau_{PS} = \pi \cdot 3 \cdot 1{,}5 \cdot 100 = 1413{,}72\,N$$

$$J = \frac{\pi}{64} d^4 = \frac{\pi}{64} \cdot 3^4 = 3{,}975\,mm^4$$

(ZMĚNA) - vethnutí oboustranné

$$j = \frac{d}{4} = 0{,}75$$

$$S = \frac{\pi d^2}{4} = \frac{\pi \cdot 3^2}{4} = 7{,}06\,mm^2$$

$$\lambda = \frac{l_{red}}{j} = \frac{60}{2 \cdot 0{,}75} = 40$$ [

$\sigma_k = 589 - 3{,}82 \cdot 40 = 436{,}2\,MPa$

$F_k = S \cdot \sigma_k = 7{,}06 \cdot 436{,}2 = 3079{,}57\,N$

$k = \frac{F_u}{F} = \frac{3079{,}57}{1413{,}72} = 2{,}178$

Vyhovuje

4.325 | $l_2 = 70\,mm$ — 1 vetknutí, 2 kloubový

$\lambda = \frac{l_{red}}{j'} = \frac{49,5}{0,75} = \underline{\underline{66}}$ $\quad l_{red} = \frac{l}{\sqrt{2}} = 49,5\,mm$

Vzpříst podle Tetmajera – legovaná ocel (jako ocel Ni)

$\sigma_{kr} = 589 - 3,82\lambda = 589 - 3,82 \cdot 66 = \underline{336,88\,MPa}$

$F_{kr} = S \cdot \sigma_{kr} = 7,06 \cdot 336,88 = 2378,37\,N$

$k = \frac{F_{kr}}{F} = \frac{2378,37}{1413,72} = \underline{\underline{1,68}}$ – nevyhovuje

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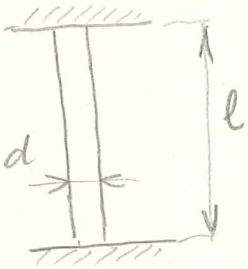
4.326 |



$F = 50\,kN$ $\quad E = 2,1 \cdot 10^5\,MPa$

$d = 80\,mm$ $\quad \sigma_u = 220\,MPa$

$l = 2\,m$ $\quad \alpha_t = 11,5 \cdot 10^{-6}\,K^{-1}$

$k = 3$ $\quad \Delta t = ?$

$J = \frac{\pi}{64} \cdot d^4 = \frac{\pi}{64} \cdot 80^4 = 2010560\,mm^4$ $\quad \underline{l_{red} = l}$

$j' = \frac{d}{4} = \frac{80}{4} = 20\,mm$

$\lambda = \frac{l_{red}}{j'} = \frac{2000}{20} = 100$ $\quad \lambda_m = \pi\sqrt{\frac{E}{\sigma_u}} = \pi\sqrt{\frac{2,1 \cdot 10^5}{220}} = \underline{97,05}$

$S = \frac{\pi d^2}{4} = \frac{\pi \cdot 80^2}{4} = 5026,5\,mm^2$ $\quad \underline{\underline{\lambda > \lambda_m}}$

Vzpříst podle Eulera

$F_{kr} = \frac{\pi^2 E J}{l_{red}^2} = \frac{\pi^2 \cdot 2,1 \cdot 10^5 \cdot 2010560}{2000^2} = 1041780\,N$

$F_c = \frac{F_{kr}}{k} = \frac{1041780}{3} = 347260\,N$

$F_t = F_c - F = 297260\,N$

$\sigma_t = \frac{F_t}{S} = \frac{297260}{5026,5} = 59,14\,MPa$

$\sigma = 59,13$

4.327 | $b = 100\,mm$ F

obr. 4.168

jeden vetknut druhý kloub

$l = 4\,m$ $\perp x$

$\lambda_K = ?$

$J_{min} = \frac{1}{12} b h^3 = \frac{b^4}{12}$

$= 0{,}08 \cdot 10^8 = 8 \cdot 10^6\,mm^4$

$S = b^2 = 10^4\,mm^2$

$i = \sqrt{\frac{J}{S}} = \sqrt{\frac{8 \cdot 10^6}{10^4}} =$

$= 10\sqrt{8} = 28{,}28\,mm$

$\lambda = \frac{l_{red}}{i} = \frac{2000}{28{,}28} = 70{,}72$

$l_{red\,x} = 1000\,mm$

$l_{red} = 1400\,mm$

$l_{red\,y} = 2000\,mm$

69,2

4.328 | 2

$F_1 = 80\,kN$ L 100 x 100 x 6

4.169

$l = 2200\,mm = l_{red\,y}$

$l_{red\,x} = 0{,}95\,l$

$\lambda = 93$

$\sigma_{Dd} = 140\,MPa$

Euler ~~$F_{kr} = \frac{\pi^2 E J}{l_{red}^2} =$~~

c

3 tab. $S = 1179\,mm^2$

$F_c = \sigma_{Dd} \cdot S$

$F = \frac{\sigma_{Dd} \cdot S}{c} = \frac{140 \cdot 1179}{1{,}644} =$

$= 100\,401\,N$

$\lambda = 90 \dots c = 1{,}59$

$\lambda = 100 \dots c = 1{,}77$

$\lambda = 93 \dots c = 1{,}644$

λ