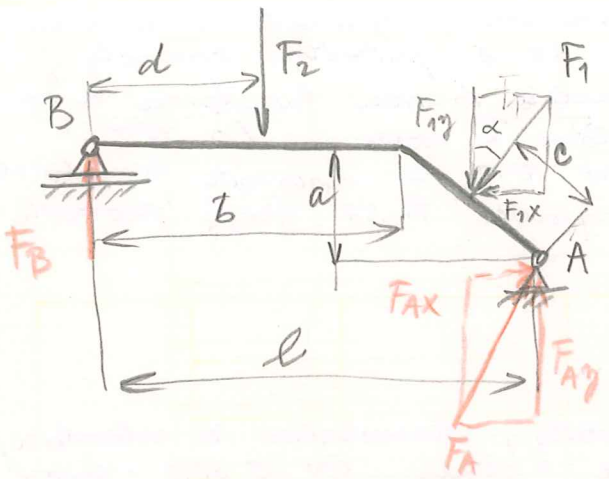
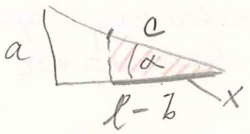
1.75



$F_1 = 270\,N$
$F_2 = 410\,N$
$a = 540\,mm$
$b = 450\,mm$
$c = 230\,mm$
$d = 260\,mm$
$l = 960\,mm$



$tg\alpha = \frac{a}{l-b} = \frac{540}{960-260} = 0{,}771$

$\alpha = 37{,}64°$

$F_{1x} = F_1 \sin\alpha = 270 \sin 37{,}64 = 164{,}8\,N$

$F_{1y} = F_1 \cos\alpha = 270 \cos 37{,}64 = 213{,}8\,N$

$x = c \cdot \cos\alpha = 230 \cdot \cos 37{,}64 = 182{,}1\,mm$

$x: \quad \Sigma F_{ix} = 0 \quad -F_{1x} + F_{Ax} = 0$

$y: \quad \Sigma F_{iy} = 0 \quad -F_B + F_2 + F_{1y} - F_{Ay} = 0$

$M_B: \quad \Sigma M_i = 0 \quad -F_2 d - F_{1y}(l-x) + F_{Ax} \cdot a + F_{Ay} \cdot l = 0$

$F_{Ax} = F_{1x} = 164{,}8\,N$

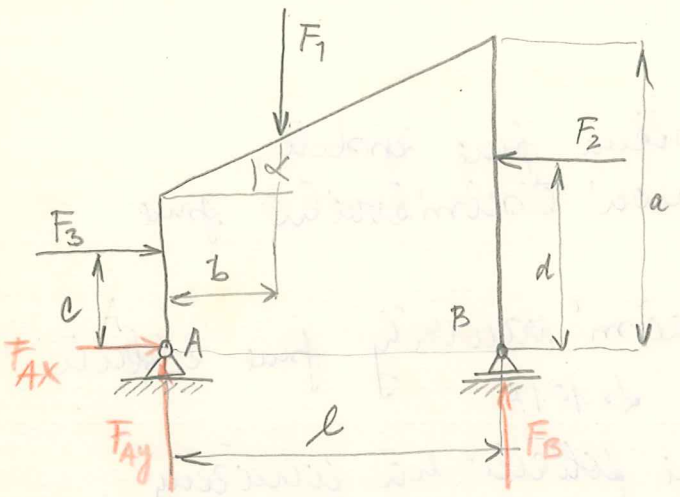
$$F_{Ay} = \frac{F_2 d + F_{1y}(l-x) - F_{Ax} \cdot a}{l} = \frac{410 \cdot 260 + 213{,}8 \cdot (960 - 182{,}1) - 164{,}8 \cdot 540}{960}$$

$$= \frac{106\,600 + 166\,315 - 88\,992}{960} \doteq 191{,}6\,N$$

$F_A = \sqrt{F_{Ax}^2 + F_{Ay}^2} = \sqrt{164{,}8^2 + 191{,}6^2} = 252{,}7\,N$

$F_B = F_2 + F_{1y} - F_{Ay} = 410 + 213{,}8 - 191{,}6 = 432{,}2\,N$

1.76



$F_1 = 820\,N$
$F_2 = 740\,N$
$F_3 = 530\,N$

$a = 1{,}9\,m$
$b = 1{,}1\,m$
$c = 0{,}3\,m$
$d = 1{,}3\,m$
$l = 1{,}7\,m$
$\alpha = 25°$

$F_A = ?$
$F_B = ?$

$x: \quad \sum F_{ix} = 0 \quad \dots \quad F_{Ax} + F_3 - F_2 = 0$

$y: \quad \sum F_{iy} = 0 \quad \dots \quad -F_{Ay} + F_1 - F_B = 0$

$M_A: \quad \sum M_i = 0 \quad \dots \quad -F_3 \cdot c - F_1 \cdot b + F_2 \cdot d + F_B \cdot l = 0$

$$F_B = \frac{F_3 c + F_1 b - F_2 d}{l} = \frac{530 \cdot 0{,}3 + 820 \cdot 1{,}1 - 740 \cdot 1{,}3}{1{,}7} =$$

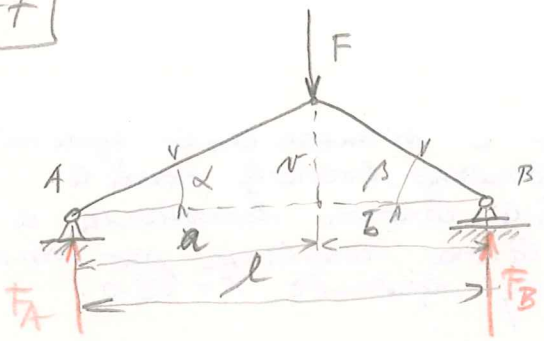
$$= \frac{159 + 902 - 962}{1{,}7} = \underline{58{,}2\,N}$$

$$F_{Ax} = F_2 - F_3 = 740 - 530 = \underline{210\,N}$$

$$F_{Ay} = F_1 - F_B = 820 - 58{,}2 = \underline{761{,}8\,N}$$

$$F_A = \sqrt{F_{Ax}^2 + F_{Bx}^2} = \sqrt{210^2 + 761{,}8^2} = \underline{790{,}2\,N}$$

1,77



$F = 240\ N$

$\alpha = 65°$

$\beta = 53°$

$l = 580\ mm$

$x: \dots = 0$

$y: \Sigma F_{iy} = 0 \dots F_A - F + F_B = 0$

$M_A: \Sigma M_i = 0 \dots F_1 a - F_B \cdot l = 0$

$F_B = F\frac{a}{l} = 240\,\frac{221,7}{580} = 91,7\ N$

$F_A = F - F_B = 240 - 91,7 = 148,7\ N$

$\frac{\nu}{a} = tg\alpha \quad \nu = a\,tg\alpha$

$\frac{\nu}{b} = tg\beta \quad b = \frac{\nu}{tg\beta} = \frac{a\,tg\alpha}{tg\beta}$

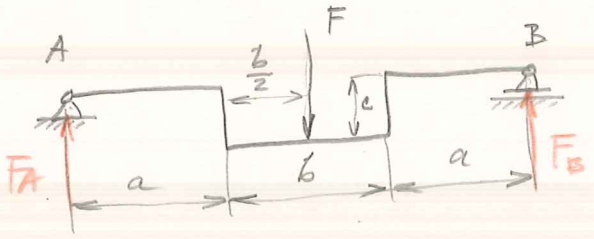
$b = l - a$

$l - a = a\frac{tg\alpha}{tg\beta}$

$l = a\left(\frac{tg\alpha}{tg\beta} + 1\right)$

$a = \frac{l}{\frac{tg\alpha}{tg\beta} + 1} = \frac{580}{\frac{2,144}{1,327} + 1} = 221,7\ mm$

1,78



$F = 8600\ N$

$a = 2,2\ m$

$b = 1,8\ m$

$c = 2,4\ m$

$x: \dots = 0$

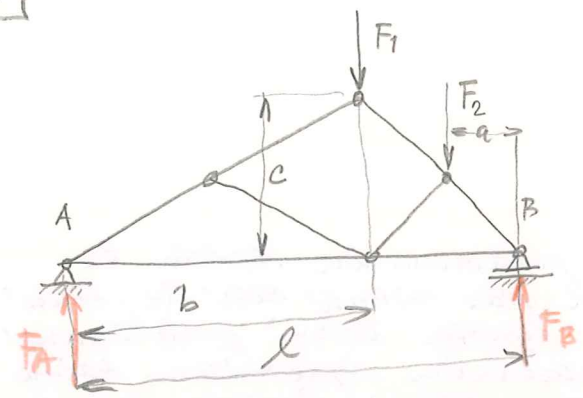
$y: \Sigma F_{iy} = 0 \quad F_A - F + F_B = 0$

$M_A: \Sigma M_i = 0 \dots F\left(a + \frac{b}{2}\right) - F_B(2a + b) = 0$

$F_B = \frac{F\left(a + \frac{b}{2}\right)}{2a + b} = 8600\,\frac{2,2 + 0,9}{4,4 + 1,8} = 4300\ N$

$F_A = F - F_B = 8600 - 4300 = 4300\ N$

1.79



$F_1 = 2600\,N$
$F_2 = 3800\,N$
$a = 0{,}8\,m$
$b = 1{,}1\,m$
$c = 0{,}7\,m$
$l = 2{,}4\,m$

$X: \quad 0$

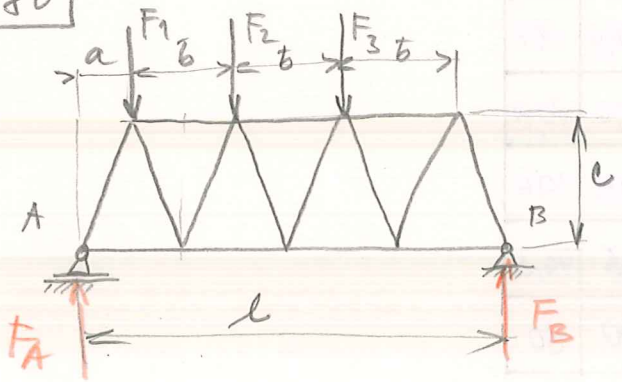
$y: \Sigma F_{iy} = 0 \quad F_A - F_1 - F_2 + F_B = 0$

$M_A: \Sigma M_i = 0 \quad F_1 \cdot b + F_2(l-a) - F_B l = 0$

$$F_B = \frac{F_1 b + F_2(l-a)}{l} = \frac{2600 \cdot 1{,}1 + 3800(2{,}4-0{,}8)}{2{,}4} = 3725\,N$$

$$F_A = F_1 + F_2 - F_B = 2600 + 3800 - 3725 = 2675\,N$$

1.80



$F_1 = 920\,N$
$F_2 = 580\,N$
$F_3 = 740\,N$
$a = 1\,m$
$b = 2\,m$
$c = 2\,m$
$l = 8\,m$

$X: \quad 0$

$y: \Sigma F_{iy} = 0 \quad -F_A + F_1 + F_2 + F_3 - F_B = 0$

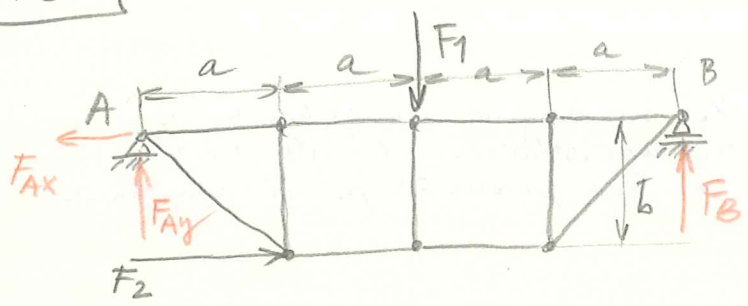
$M_A: \Sigma M_i = 0 \quad F_1 \cdot a + F_2(a+b) + F_3(a+2b) - F_B \cdot l = 0$

$$F_B = \frac{F_1 \cdot a + F_2(a+b) + F_3(a+2b)}{l} = \frac{920 \cdot 1 + 580(1+2) + 740(1+2 \cdot 2)}{8}$$

$$= 795\,N$$

$$F_A = F_1 + F_2 + F_3 - F_B = 920 + 580 + 740 - 795 = 1445\,N$$

1.81



$F_1 = 12\,600\,N$
$F_2 = 8500\,N$
$a = 2{,}3\,m$
$b = 3{,}7\,m$

$X: \quad -\Sigma F_{ix} = 0 \quad \dots \quad -F_{Ax} + F_2 = 0$

$y: \quad -\Sigma F_{iy} = 0 \quad \dots \quad F_{Ay} - F_1 + F_B = 0$

$M_A \quad -\Sigma M_i = 0 \quad \dots \quad F_1 \cdot 2a - F_2 \cdot b - F_B \cdot 4a = 0$

$$F_B = \frac{F_1 \cdot 2a - F_2 b}{4a} = \frac{12600 \cdot 2 \cdot 2{,}3 - 8500 \cdot 3{,}7}{4 \cdot 2{,}3} = \underline{2881{,}5\,N}$$

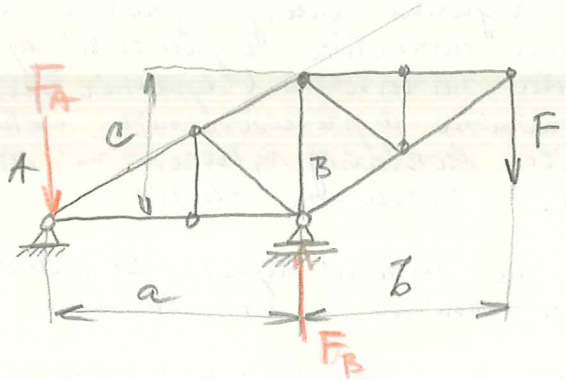
$$F_{Ax} = F_2 = 8500\,N$$

$$F_{Ay} = F_1 - F_B = 12600 - 2881{,}5 = \underline{9718{,}5\,N}$$

$$F_A = \sqrt{F_{Ax}^2 + F_{Ay}^2} = \sqrt{2881{,}5^2 + 9718{,}5^2} = \underline{10\,136{,}6\,N}$$

1.82

$F = 7200\,N$
$a = 1{,}6\,m$
$b = 1{,}3\,m$
$c = 0{,}3\,m$



$X: \quad \dots \quad 0$

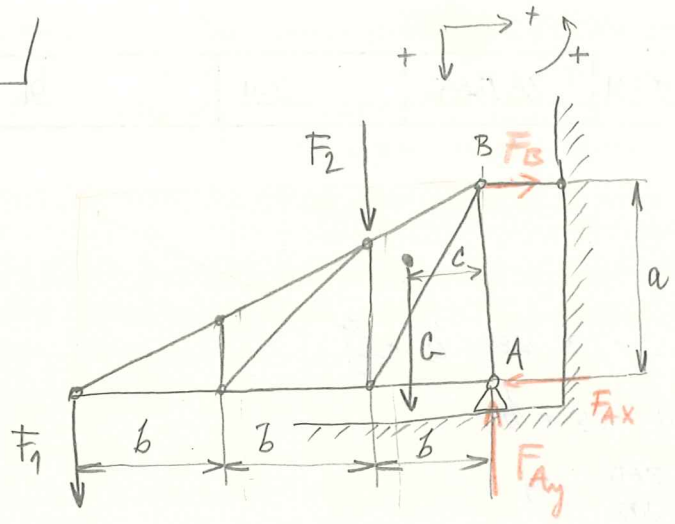
$y: \quad \Sigma F_{iy} = 0 \quad \dots \quad -F_A + F_B - F = 0$

$M_A: \quad \Sigma M_i = 0 \quad \dots \quad F_B \cdot a - F(a+b) = 0$

$$F_B = F\frac{a+b}{a} = 7200\,\frac{2{,}9}{1{,}6} = \underline{13\,050\,N}$$

$$F_A = F_B - F = 13050 - 7200 = \underline{5850\,N}$$

1,83



$F_1 = 1600\,N$

$F_2 = 1400\,N$

$G = 1100\,N$

$a = 1,3\,m$

$b = 0,9\,m$

$c = 0,7\,m$

$x: \Sigma F_{ix} = 0$ —— $F_B - F_{Ax} = 0$

$y: \Sigma F_{iy} = 0$ —— $F_1 + F_2 + G - F_{Ay} = 0$

$M: \Sigma M_i = 0$ —— $F_1 \cdot 3b + F_2 \cdot b + G \cdot c - F_B \cdot a = 0$

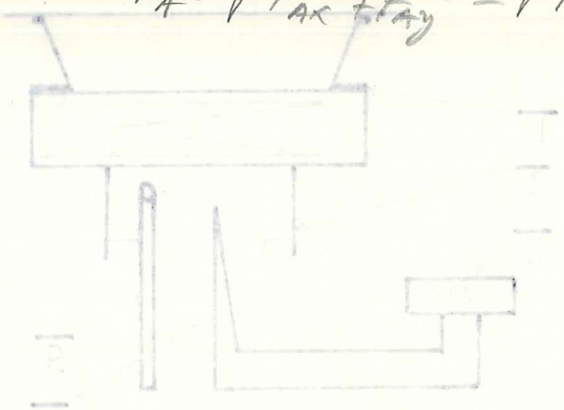
$$F_B = \frac{F_1 3b + F_2 b + G \cdot c}{a} = \frac{1600 \cdot 3 \cdot 0,9 + 1400 \cdot 0,9 + 1100 \cdot 0,7}{1,3} =$$

$$= \frac{4320 + 1260 + 770}{1,3} = \underline{4884,6\,N}$$

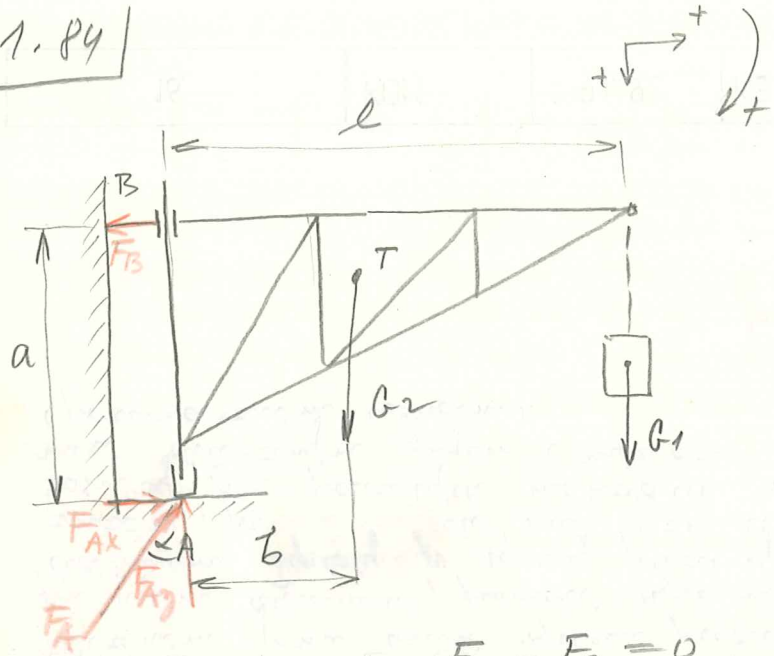
$F_{Ax} = F_B = \underline{4884,6\,N}$

$F_{Ay} = F_1 + F_2 + G = 1600 + 1400 + 1100 = \underline{4100\,N}$

$F_A = \sqrt{F_{Ax}^2 + F_{Ay}^2} = \sqrt{4884,6^2 + 4100^2} = \underline{6377,2\,N}$



1.84



$G_1 = 7600\,N$

$G_2 = 3900\,N$

$a = 4{,}4\,m$

$b = 1{,}3\,m$

$l = 3{,}8\,m$

X: $\Sigma F_{ix} = 0$ ... $F_{AX} - F_B = 0$

y: $\Sigma F_{iy} = 0$ ... $-F_{AY} + G_2 + G_1 = 0$

M: $\Sigma M_i = 0$ ... $F_B \cdot a - G_2 \cdot b - G_1 \cdot l = 0$

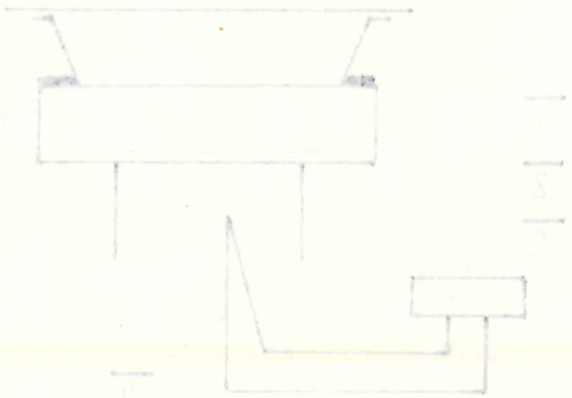
$$F_B = \frac{G_2 \cdot b + G_1 \cdot l}{a} = \frac{3900 \cdot 1{,}3 + 7600 \cdot 3{,}8}{4{,}4} = \underline{7715{,}9\,N}$$

$$F_{AX} = F_B = \underline{7715{,}9\,N}$$

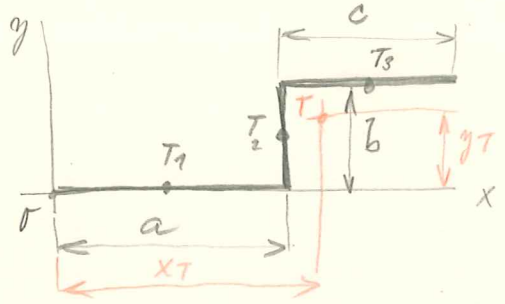
$$F_{AY} = G_2 + G_1 = 3900 + 7600 = \underline{11500\,N}$$

$$F_A = \sqrt{F_{AX}^2 + F_{AY}^2} = \sqrt{7715{,}9^2 + 11500^2} = \underline{13848{,}6\,N}$$

$$tg\,\alpha = \frac{F_{AX}}{F_{AY}} = \frac{7715{,}9}{11500} = \underline{0{,}6709} \qquad \underline{\alpha = 33{,}85^\circ}$$



1.85 Určete polohu těžiště



$a = 180\ mm$
$b = 220\ mm$
$c = 340\ mm$

$L = a + b + c = 180 + 220 + 340 = 740\ mm$

$x_1 = \frac{a}{2} = \frac{180}{2} = 90\ mm$

$x_2 = a = 180\ mm$

$x_3 = a + \frac{c}{2} = 180 + 170 = 350\ mm$

$$x_T = \frac{a \cdot x_1 + b\, x_2 + c\, x_3}{L} = \frac{180 \cdot 90 + 220 \cdot 180 + 340 \cdot 350}{740} = 236{,}2\ mm$$

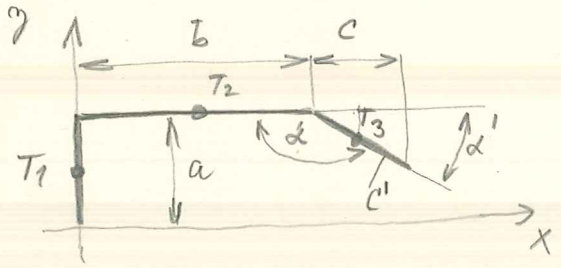
$y_1 = 0$

$y_2 = \frac{b}{2} = 110\ mm$

$y_3 = b = 220\ mm$

$$y_T = \frac{a \cdot y_1 + b \cdot y_2 + c \cdot y_3}{L} = \frac{0 + 220 \cdot 110 + 340 \cdot 220}{740} = 133{,}7\ mm$$

1.86



$a = 520\ mm$
$b = 360\ mm$
$c = 140\ mm$
$\alpha = 135°$

$L = a + b + c' = 520 + 360 + 198 = 1078\ mm$

$\alpha' = 180 - \alpha = 180 - 135° = 45°$

$c' = \frac{c}{\cos\alpha'} = \frac{140}{\cos 45°} = 198\ mm$

$x_1 = 0$

$x_2 = \frac{b}{2} = \frac{360}{2} = 180\ mm$

$x_3 = b + \frac{c}{2} = 360 + \frac{140}{2} = 430\ mm$

$$x_T = \frac{a \cdot x_1 + b \cdot x_2 + c' \cdot x_3}{L} = \frac{0 + 360 \cdot 180 + 198 \cdot 430}{1078} = 139{,}\ mm$$

$y_1 = \frac{a}{2} = \frac{520}{2} = 260\ mm$

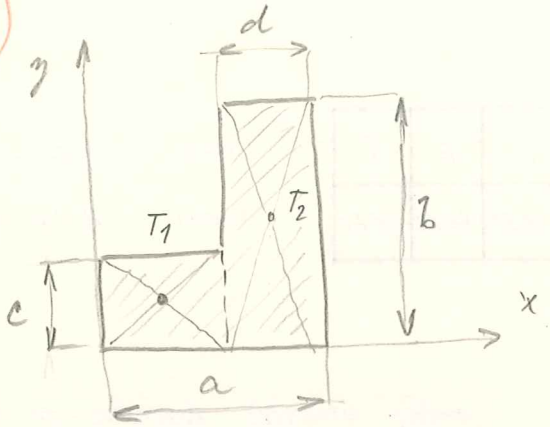
$y_2 = a = 520\ mm$

$y_3 = a - \frac{c}{2} = 520 - \frac{140}{2} = 450\ mm$

$$y_T = \frac{a \cdot y_1 + b \cdot y_2 + c' \cdot y_3}{L} = \frac{520 \cdot 260 + 360 \cdot 520 + 198 \cdot 450}{1078} = 382\ mm$$

135 200

1.89



$a = 340\ mm$
$b = 450\ mm$
$c = 110\ mm$
$d = 80\ mm$

$S_1 = c(a-d) = 110 \cdot (340-80) = 28600\ mm^2$
$S_2 = d \cdot b = 80 \cdot 450 = 36000\ mm^2$
$S = S_1 + S_2 = 64600\ mm^2$

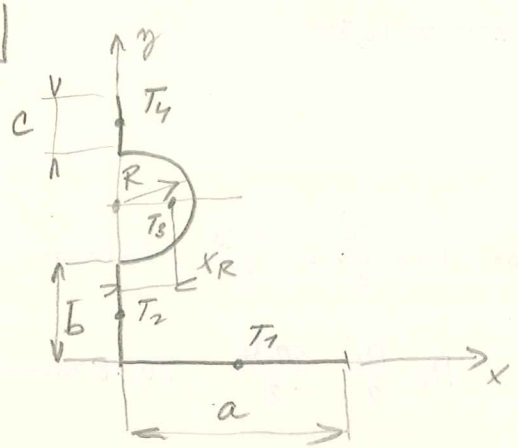
$x_1 = \frac{a-d}{2} = \frac{340-80}{2} = 130\ mm$
$x_2 = a - \frac{d}{2} = 340 - \frac{80}{2} = 300\ mm$

$y_1 = \frac{c}{2} = \frac{110}{2} = 55\ mm$
$y_2 = \frac{b}{2} = \frac{450}{2} = 225\ mm$

$$x_T = \frac{S_1 x_1 + S_2 x_2}{S} = \frac{28600 \cdot 130 + 36000 \cdot 300}{64600} = 224{,}7\ mm$$

$$y_T = \frac{S_1 y_1 + S_2 y_2}{S} = \frac{28600 \cdot 55 + 36000 \cdot 225}{64600} = 149{,}7\ mm$$

1.87



$a = 360\,mm$

$b = 220\,mm$

$c = 180\,mm$

$R = 130\,mm$

$\pi = \pi R = \pi \cdot 130 = 408\,mm$

$L = a + b + \pi R + c = 360 + 220 + \pi \cdot 130 + 180$

$= \underline{1168\,mm}$

$x_1 = \frac{a}{2} = \frac{360}{2} = 180\,mm$

$x_2 = 0$

$x_R = 0{,}64\,R = 83{,}2\,mm$

$x_3 = 0$

$$x_T = \frac{a \cdot x_1 + b x_2 + \pi \cdot x_R + c \cdot x_3}{L} = \frac{360 \cdot 180 + 0 + 408 \cdot 83{,}2 + 0}{1168} = \underline{84{,}5\,mm}$$

$y_1 = 0$

$y_2 = \frac{b}{2} = \frac{220}{2} = 110\,mm$

$y_R = b + R = 220 + 130 = 350\,mm$

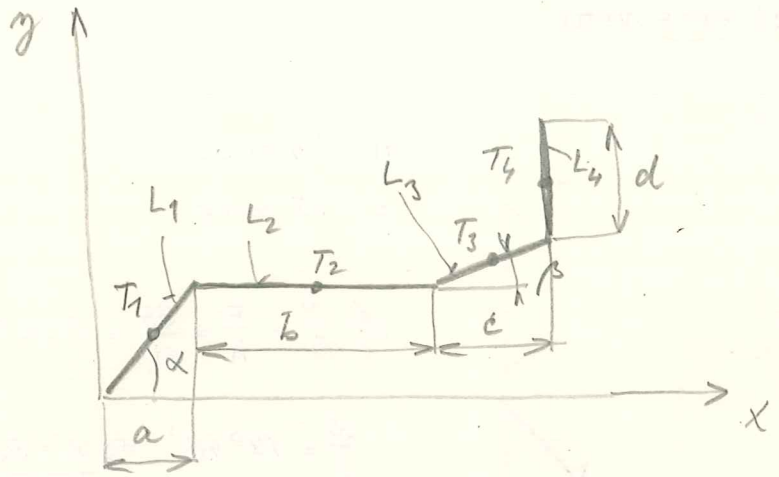
$y_3 = b + 2R + \frac{c}{2} = 220 + 2 \cdot 130 + \frac{180}{2} = 570\,mm$

24200

$$y_T = \frac{a \cdot y_1 + b \cdot y_2 + \pi \cdot y_R + c\, y_3}{L} = \frac{0 + 220 \cdot 110 + 408 \cdot 350 + 180 \cdot 570}{1168} =$$

$= \underline{230{,}8\,mm}$

1.88



$a = 150\ mm$
$b = 240\ mm$
$c = 180\ mm$
$d = 220\ mm$
$\alpha = 60°$
$\beta = 30°$

$$L_1 = \frac{a}{\cos\alpha} = \frac{150}{\cos 60°} = 300\ mm$$

$$L_2 = b = 240\ mm$$

$$L_3 = \frac{c}{\cos\beta} = \frac{180}{\cos 30°} = 207{,}8\ mm$$

$$L_4 = d = 220\ mm$$

$$L = L_1 + L_2 + L_3 + L_4 =$$
$$= 300 + 240 + 207{,}8 + 220 = \underline{967{,}8\ mm}$$

$$x_1 = \frac{a}{2} = \frac{150}{2} = 75\ mm$$

$$x_2 = \frac{b}{2} + a = \frac{240}{2} + 150 = 120 + 150 = 270\ mm$$

$$x_3 = a + b + \frac{c}{2} = 150 + 240 + \frac{180}{2} = 480\ mm$$

$$x_4 = a + b + c = 150 + 240 + 180 = 570\ mm$$

$$x_T = \frac{L_1 \cdot x_1 + L_2 \cdot x_2 + L_3 \cdot x_3 + L_4 \cdot x_4}{L} = \frac{300 \cdot 75 + 240 \cdot 270 + 207{,}8 \cdot 480 + 220 \cdot 5}{742{,}8}$$

$$= \frac{22500 + 64800 + 99744 + 125400}{967{,}8} = \underline{322{,}8\ mm}$$

$\frac{2y_1}{\frac{a}{2}} = tg\alpha$

$$y_1 = \frac{a}{2} tg\alpha = \frac{150}{2} tg\,60° = 129{,}9\ mm$$

$$y_2 = a\, tg\alpha = 150 \cdot tg\,60° = 259{,}8\ mm$$

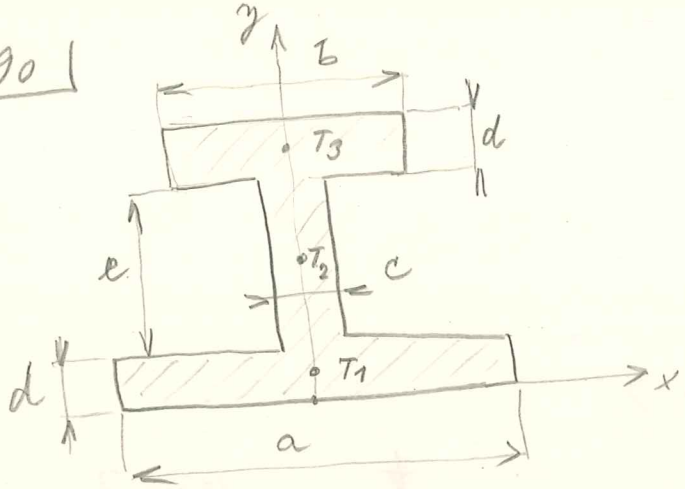
$$y_3 = y_2 + \frac{c}{2} tg\beta = 259{,}3 + \frac{180}{2} tg\,30° = 311{,}2\ mm$$

$$y_4 = y_2 + c\, tg\beta + \frac{d}{2} = 259{,}8 + 180\, tg\,30° + \frac{220}{2} = 473{,}7\ mm$$

$$y_T = \frac{L_1 y_1 + L_2 y_2 + L_3 y_3 + L_4 y_4}{L} = \frac{300 \cdot 129{,}9 + 240 \cdot 259{,}8 + 207{,}8 \cdot 311{,}2 + 220 \cdot 473{,}}{967{,}8}$$

$$= \frac{38970 + 62352 + 64667 + 104214}{967{,}8} = \underline{279{,}2\ mm}$$

1.90



$a = 480\ mm$
$b = 340\ mm$
$c = 80\ mm$
$d = 100\ mm$
$e = 450\ mm$

$S_1 = a \cdot d = 480 \cdot 100 = 48000\ mm^2$
$S_2 = c \cdot e = 80 \cdot 450 = 36000\ mm^2$
$S_3 = b \cdot d = 340 \cdot 100 = 34000\ mm^2$

$S = S_1 + S_2 + S_3 = 48000 + 36000 + 34000 = \underline{118000\ mm^2}$

$y_1 = \frac{d}{2} = \frac{100}{2} = 50\ mm$

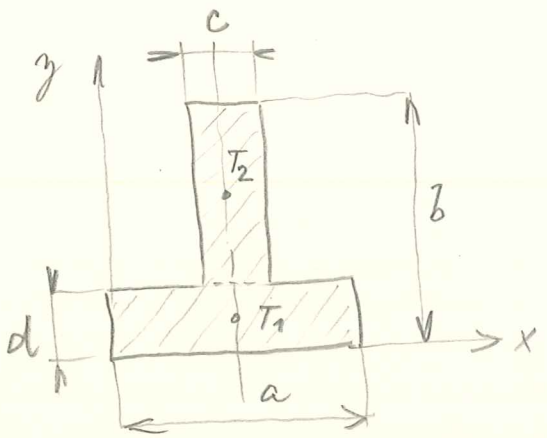
$y_2 = d + \frac{e}{2} = 100 + \frac{450}{2} = 325\ mm$

$y_3 = d + e + \frac{d}{2} = 100 + 450 + 50 = 600\ mm$

$$y_T = \frac{S_1 \cdot y_1 + S_2 y_2 + S_3 y_3}{S} = \frac{48000 \cdot 50 + 36000 \cdot 325 + 34000 \cdot 600}{118000}$$

$$= \underline{292{,}3\ mm}$$

1.91



$a = 480\ mm$

$b = 540\ mm$

$c = 80\ mm$

$d = 100\ mm$

$S_1 = a \cdot d = 480 \cdot 100 = 48\,000\ mm^2$

$S_2 = (b-d)\,c = (540-100)\,80 = 35\,200\ mm^2$

$S = S_1 + S_2 = 48000 + 35200 = 83\,200\ mm^2$

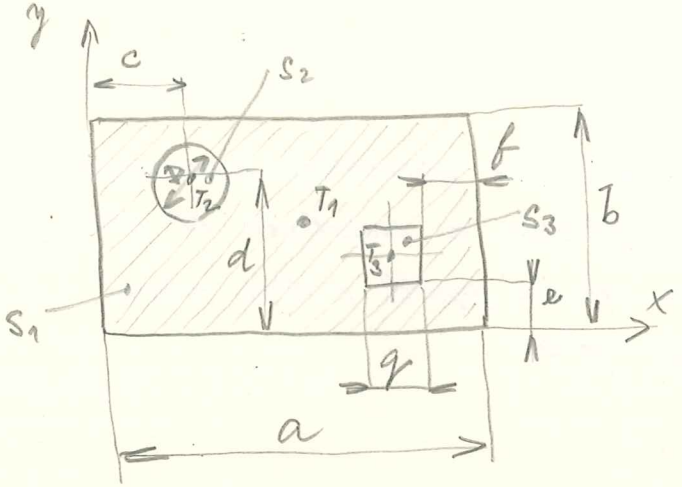
$$y_1 = \frac{d}{2} = \frac{100}{2} = 50\ mm$$

$$y_2 = \frac{b-d}{2} + d = \frac{540-100}{2} + 100 = 320\ mm$$

$$x_y = \frac{S_1 y_1 + S_2 y_2}{S} = \frac{48000 \cdot 50 + 35200 \cdot 320}{83200} = 164{,}2\ mm$$

$$x_T = \frac{a}{2} = 240\ mm$$

1.92



$a = 740$ mm $D = 110$ mm
$b = 580$ mm
$c = 150$ mm
$d = 380$ mm
$e = 80$ mm
$f = 160$ mm
$q = 230$ mm

$S_1 = a \cdot b = 740 \cdot 580 = 429200\ mm^2$

$S_2 = \frac{\pi D^2}{4} = \frac{\pi \cdot 110^2}{4} = 9503{,}3\ mm^2$

$S_3 = q^2 = 230^2 = 52900\ mm^2$

$S = S_1 - S_2 - S_3 = 429200 - 9503{,}3 - 52900 = \underline{366796{,}7\ mm^2}$

$x_1 = \frac{a}{2} = \frac{740}{2} = 370\ mm$

$x_2 = c = 150\ mm$

$x_3 = a - \left(f + \frac{q}{2}\right) = 740 - \left(160 + \frac{230}{2}\right) = 465\ mm$

$$x_T = \frac{S_1 \cdot x_1 - S_2 x_2 - S_3 x_3}{S} = \frac{429200 \cdot 370 - 9503{,}3 \cdot 150 - 52900 \cdot 465}{366796{,}7}$$

$$= \underline{361{,}9\ mm}$$

$y_1 = \frac{b}{2} = \frac{580}{2} = 290\ mm$

$y_2 = d = 380\ mm$

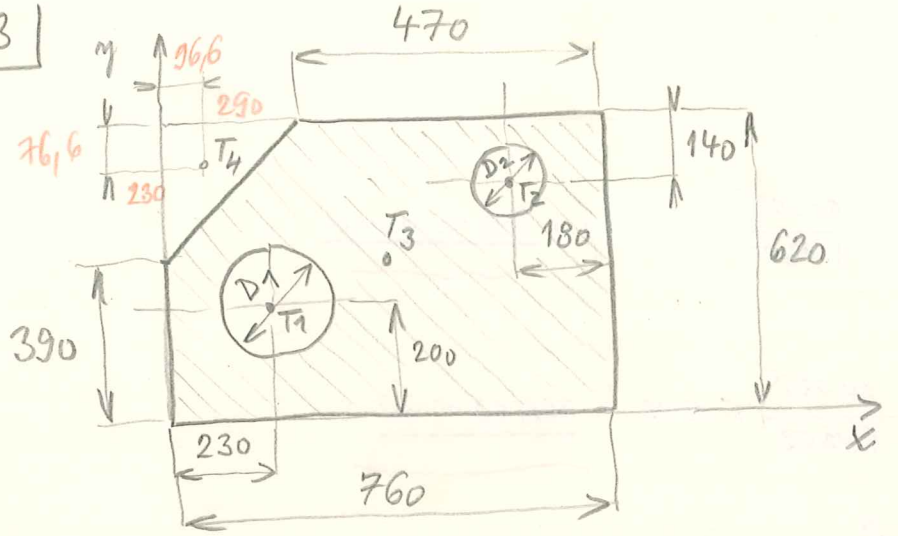
$y_3 = e + \frac{q}{2} = 80 + \frac{230}{2} = 195\ mm$

195

$$y_T = \frac{S_1 y_1 - S_2 y_2 - S_3 y_3}{S} = \frac{429200 \cdot 290 - 9503{,}3 \cdot 380 - 52900 \cdot}{366796{,}7}$$

$$= \underline{301{,}3\ mm}$$

1.93



$D_1 = 340\ mm$

$D_2 = 210\ mm$

$S_1 = \frac{\pi D_1^2}{4} = \frac{\pi \cdot 340^2}{4} = 90792\ mm^2$

$S_2 = \frac{\pi D_2^2}{4} = \frac{\pi \cdot 210^2}{4} = 34636\ mm^2$

$S_3 = 760 \cdot 620 = 471200\ mm^2$

$S_4 = 290 \cdot \frac{230}{2} = 33350\ mm^2$

$S = S_3 - (S_1 + S_2 + S_4) =$

$= 471200 - (90792 + 34636 + 33350)$

$= \underline{312422\ mm^2}$

$x_1 = 230\ mm$

$x_2 = 760 - 180 = 580\ mm$

$x_3 = \frac{760}{2} = 380\ mm$

$x_4 = 96,6\ mm$

$y_1 = 200\ mm$

$y_2 = 620 - 140 = 480\ mm$

$y_3 = \frac{620}{2} = 310\ mm$

$y_4 = 620 - 76,6 = 543,4\ mm$

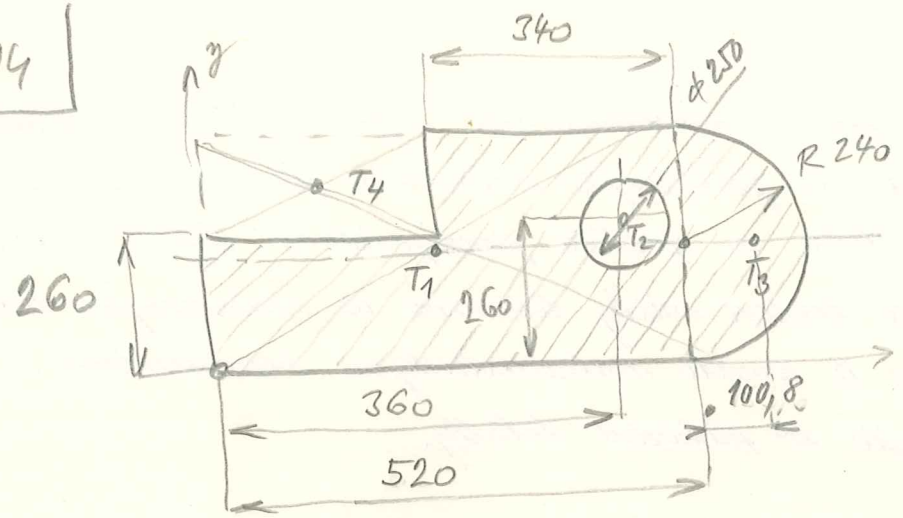
$$x_T = \frac{S_3 x_3 - (S_1 x_1 + S_2 x_2 + S_4 x_4)}{S} = \frac{471200 \cdot 380 - (90792 \cdot 230 + 34636 \cdot 580 + 33350 \cdot 96,6)}{312422}$$

$$= \underline{431,6\ mm}$$

$$y_T = \frac{S_3 y_3 - (S_1 y_1 + S_2 y_2 + S_4 y_4)}{S} = \frac{471200 \cdot 310 - (90792 \cdot 200 + 34636 \cdot 480 + 33350 \cdot 543,4)}{312422}$$

$$= \underline{298,2\ mm}$$

1.94



240 . 0,42 =

$S_1 = 520 \cdot 480 = 249600\ mm^2$

$S_2 = \frac{\pi \cdot 250^2}{4} = 49087\ mm^2$

$S_3 = \frac{1}{2}\,\frac{\pi \cdot 480^2}{4} = 90477{,}8\ mm^2$

$S_4 = (520-340)(480-260) = 39600\ mm^2$

$S = S_1 - S_2 - S_4 + S_3 =$

$= 249600 - 49087 - 39600 + 90477{,}8 =$

$= \underline{\underline{251390{,}8\ mm^2}}$

$x_1 = \frac{520}{2} = 260\ mm$

$x_2 = 360\ mm$

$x_3 = 520 + 100{,}8 = 620{,}8\ mm$

$x_4 = \frac{520-340}{2} = 90\ mm$

$y_1 = 240\ mm$

$y_2 = 260\ mm$

$y_3 = 240\ mm$

$y_4 = 260 + \frac{480-260}{2} = 370\ mm$

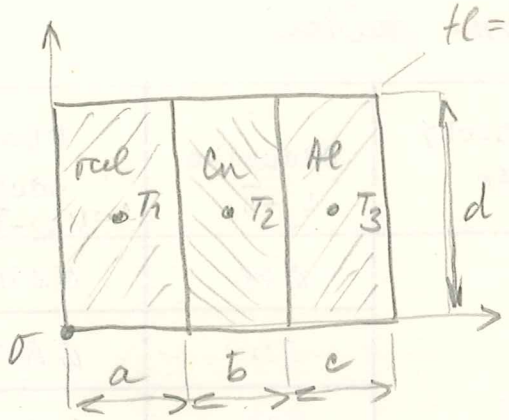
$$x_T = \frac{S_1 x_1 + S_3 x_3 - S_2 x_2 - S_4 x_4}{S} =$$

$$= \frac{249600 \cdot 260 + 90477{,}8 \cdot 620{,}8 - 49087 \cdot 360 - 39600 \cdot 90}{251390{,}8} = \underline{\underline{397{,}1\ mm}}$$

$$y_T = \frac{S_1 y_1 + S_3 y_3 - S_2 y_2 - S_4 y_4}{S} =$$

$$= \frac{249600 \cdot 240 + 90477{,}8 \cdot 240 - 49087 \cdot 260 - 39600 \cdot 370}{251390{,}8} = \underline{\underline{215{,}6\ mm}}$$

1.95



$a = 16\,cm$
$b = 22\,cm$
$c = 18\,cm$
$d = 35\,cm$
$t = 2\,cm$

$\rho_1 = 7800$	rul
$\rho_2 = 8900$	Cu
$\rho_3 = 2700$	Al

$S_1' = S_1 \rho_1 = 0,16 \cdot 0,35 \cdot 7800 = 436,8$

$S_2' = S_2 \rho_2 = 0,22 \cdot 0,35 \cdot 8900 = 685,3$

$S_3' = S_3 \rho_3 = 0,18 \cdot 0,35 \cdot 2700 = 170,1 \quad [m^2 \cdot kg \cdot m^{-3} = kg \cdot m^{-1}]$

$S' = S_1' + S_2' + S_3' = 436,8 + 685,3 + 170,1 = \underline{1292,2}\ kg\,m^{-1}$

$x_1 = \frac{a}{2} = \frac{0,16}{2} = 0,08\,m$

$x_2 = a + \frac{b}{2} = 0,16 + \frac{0,22}{2} = 0,27\,m$

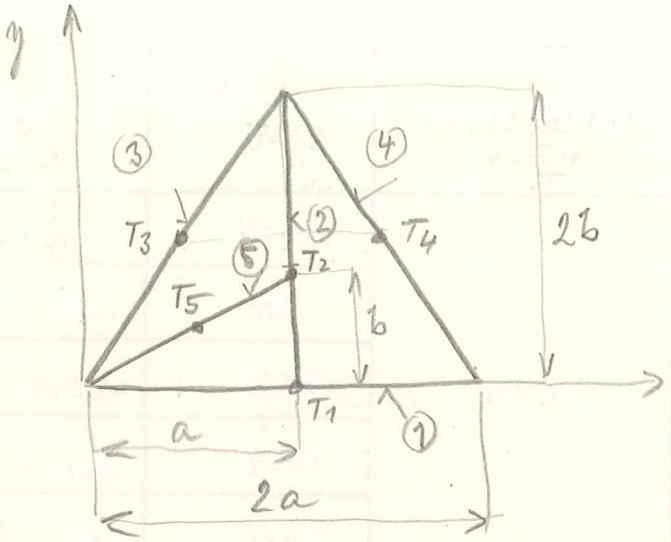
$x_3 = a + b + \frac{c}{2} = 0,16 + 0,22 + \frac{0,18}{2} = 0,47\,m$

$$x_T = \frac{S_1' \cdot x_1 + S_2' x_2 + S_3' x_3}{S'} = \frac{436,8 \cdot 0,08 + 685,3 \cdot 0,27 + 170,1 \cdot 0,47}{1292,2}$$

$= 0,232\,m = \underline{23,2\,cm}$

$z_T = \frac{d}{2} = \frac{35}{2} = \underline{17,5\,cm}$

1.96



$a = 4\,m$
$b = 3\,m$

$L_1 = 2a = 2 \cdot 4 = 8\,m$
$L_2 = 2b = 2 \cdot 3 = 6\,m$
$L_3 = \sqrt{\left(\frac{L_1}{2}\right)^2 + L_2^2} = \sqrt{a^2 + (2b)^2} = \sqrt{4^2 + 6^2} = 7{,}21\,m$
$L_4 = 7{,}21$
$L_5 = \sqrt{a^2 + b^2} = \sqrt{4^2 + 3^2} = 5\,m$

$L = L_1 + L_2 + L_3 + L_4 + L_5 = 8 + 6 + 7{,}21 + 7{,}21 + 5 = 33{,}42\,m$

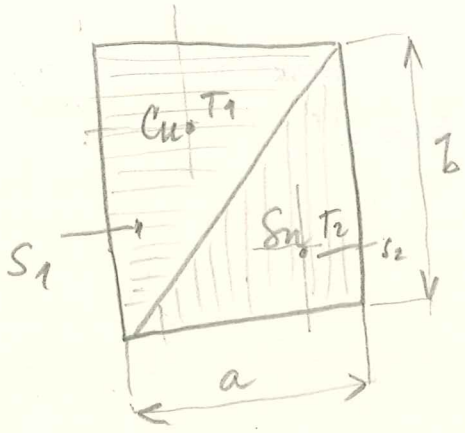
$x_1 = a = 4\,m$
$x_2 = a = 4\,m$
$x_3 = \frac{a}{2} = 2\,m$
$x_4 = a + \frac{a}{2} = 6\,m$
$x_5 = \frac{a}{2} = 2\,m$

$$x_T = \frac{L_1 x_1 + L_2 x_2 + L_3 x_3 + L_4 x_4 + L_5 x_5}{L} = \frac{8 \cdot 4 + 6 \cdot 4 + 7{,}21 \cdot 2 + 7{,}21 \cdot 6 + 5 \cdot 2}{33{,}42} = 3{,}7\,m$$

$y_1 = 0$
$y_2 = b = 3\,m$
$y_3 = \frac{2b}{2} = b = 3\,m$
$y_4 = 3\,m$
$y_5 = \frac{b}{2} = 1{,}5\,m$

$$y_T = \frac{L_1 y_1 + L_2 y_2 + L_3 y_3 + L_4 y_4 + L_5 y_5}{L} = \frac{0 + 6 \cdot 3 + 7{,}21 \cdot 3 + 7{,}21 \cdot 3 + 5 \cdot 1{,}5}{33{,}42} = 2{,}06\,m$$

1.97



Cu --- $\rho_1 = 8930$

Sn --- $\rho_2 = 7280$

$a = 960\ mm$

$b = 1100\ mm$

$S_1' = S_1 \cdot \rho_1 = \frac{1}{2}\, 0{,}96 \cdot 1{,}1 \cdot 8930 = 4715$

$S_2' = S_2 \rho_2 = \frac{1}{2}\, 0{,}96 \cdot 1{,}1 \cdot 7280 = 3843{,}8$

$S = S_1' + S_2' = 8558{,}8$

$x_1 = \frac{a}{3} = 320\ mm$ | $y_1 = \frac{2}{3} b = 733{,}3\ mm$

$x_2 = \frac{2}{3} a = 640\ mm$ | $y_2 = \frac{b}{3} = 366{,}6\ mm$

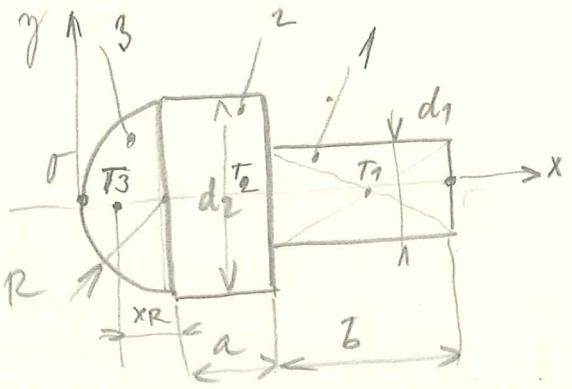
$$x_T = \frac{S_1' x_1 + S_2' x_2}{S'} = \frac{4715 \cdot 0{,}32 + 3843 \cdot 0{,}64}{8558{,}8} = 0{,}4636\ m$$

$$= \underline{463{,}6\ mm}$$

$$y_T = \frac{S_1' \cdot y_1 + S_2' y_2}{S'} = \frac{4715 \cdot 0{,}7333 + 3843{,}8 \cdot 0{,}3666}{8558{,}8} =$$

$$= 0{,}568\ m = \underline{568\ mm}$$

1.98



$d_1 = 300\ mm$

$d_2 = 500\ mm$

$a = 250\ mm$

$b = 400\ mm$

$R = 250\ mm$

$x_R = 0{,}42R$

$S_1 = d_1 b = 300 \cdot 400 = 120000\ mm^2$

$S_2 = d_2 a = 500 \cdot 250 = 125000\ mm^2$

$S_3 = \frac{\pi R^2}{2} = \pi \frac{250^2}{2} = 98174\ mm^2$

$S = S_1 + S_2 + S_3 = 120000 + 125000 + 98174 = 343174\ mm^2$

$x_1 = R + a + \frac{b}{2} = 250 + 250 + 200 = 700\ mm$

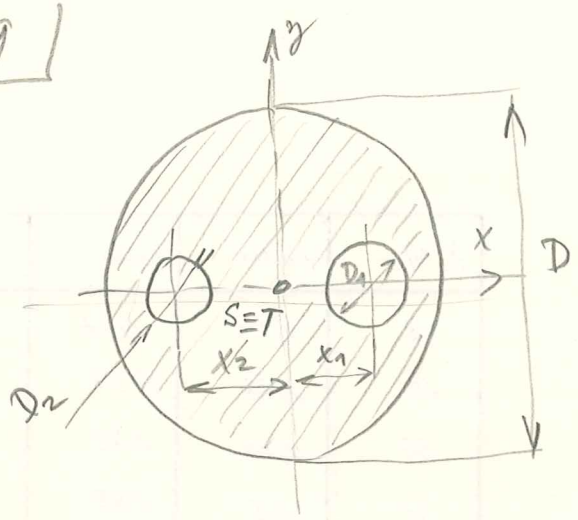
$x_2 = R + \frac{a}{2} = 250 + \frac{250}{2} = 375\ mm$

$x_3 = R - 0{,}42R = R(1 - 0{,}42) = 250 \cdot 0{,}58 = 145\ mm$

$$x_T = \frac{S_1 x_1 + S_2 x_2 + S_3 x_3}{S} = \frac{120000 \cdot 700 + 125000 \cdot 375 + 98174 \cdot 145}{343174} =$$

$= 422{,}8\ mm$?

1.99



$D = 1400\ mm$

$S = 40\ mm$

$D_1 = 320\ mm$

$X_1 = 260\ mm$

$X_2 = ?$

$D_2 = 270\ mm$

$T \equiv S$

$$S_1 X_1 = S_2 X_2$$

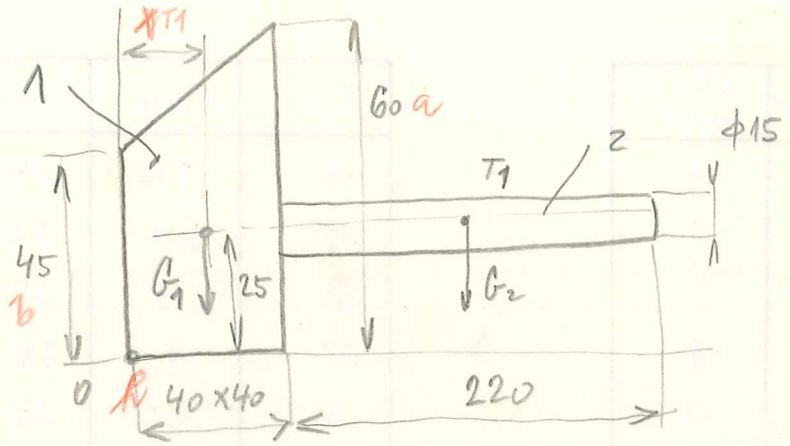
$$\frac{\pi D_1^2}{4} \cdot X_1 = \frac{\pi D_2^2}{4} X_2$$

$$X_2 = X_1 \frac{D_1^2}{D_2^2} = X_1 \left(\frac{D_1}{D_2}\right)^2 = 260 \left(\frac{320}{270}\right)^2 = \underline{\underline{365{,}2\ mm}}$$

1.100

a) celé dřevěné $\rho_0 = 750\ kg\,m^{-3}$

b) kladivo ocel, násada dřevěná — $\rho = 7800\ kg\,m^{-3}$



$$x_{T1} = \frac{h(2a+b)}{3(a+b)}$$

$$x_{T1} = \frac{h(2a+b)}{3(a+b)} = \frac{40(2\cdot 60+45)}{3(60+45)} = 20{,}95\ mm$$

a) $G_1 = mg = V\rho_0 g = \frac{0{,}045+0{,}060}{2}\cdot 0{,}04\cdot 0{,}04\cdot 750\cdot 9{,}81 = 0{,}618\ N$

$G_2 = \frac{\pi d^2}{4}\cdot l\cdot \rho g = \frac{\pi\cdot 0{,}015^2}{4}\cdot 0{,}22\cdot 750\cdot 9{,}81 = 0{,}286\ N$

$G = G_1 + G_2 = 0{,}904\ N$

$x_1 = 20{,}95\ mm$

$x_2 = 40 + 110 = 150\ mm$

$x_T \cdot G = G_1 x_1 + G_2 \cdot x_2$

$x_T = \frac{G_1 x_1 + G_2 x_2}{G} =$

$= \frac{0{,}618\cdot 20{,}95 + 0{,}286\cdot 150}{0{,}904} = 61{,}77\ mm$

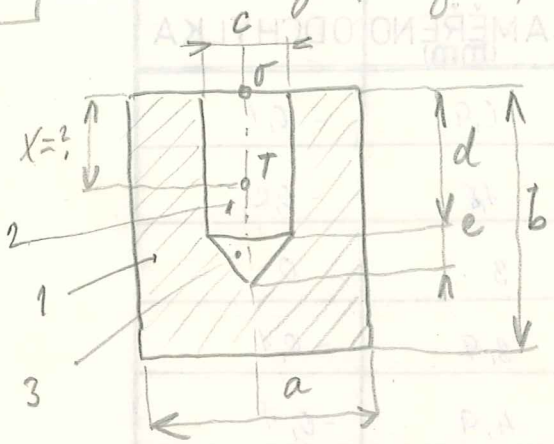
b) $G_1 = \frac{0{,}045+0{,}06}{2}\cdot 0{,}04\cdot 0{,}04\cdot 7800\cdot 9{,}81 = 6{,}427\ N$

$G_2 = 0{,}286$ $\quad G = 6{,}713\ N$

$$x_T = \frac{G_1 x_1 + G_2 x_2}{G} = \frac{6{,}427\cdot 20{,}95 + 0{,}286\cdot 150}{6{,}713} = 26{,}44\ mm$$

y_T — nepodstatné (celk. pracný výpočet)

1.101 | Určete graficky i početně x_T



$a = 60\ mm$
$b = 130\ mm$
$c = 40\ mm$
$d = 80\ mm$
$e = 30\ mm$

$S_1 = a\,b = 60 \cdot 130 = 7800\ mm^2$
$S_2 = c\,d = 40 \cdot 80 = 3200\ mm^2$
$S_3 = \frac{1}{2}\,c\,e = \frac{1}{2}\,40 \cdot 30 = 600\ mm^2$

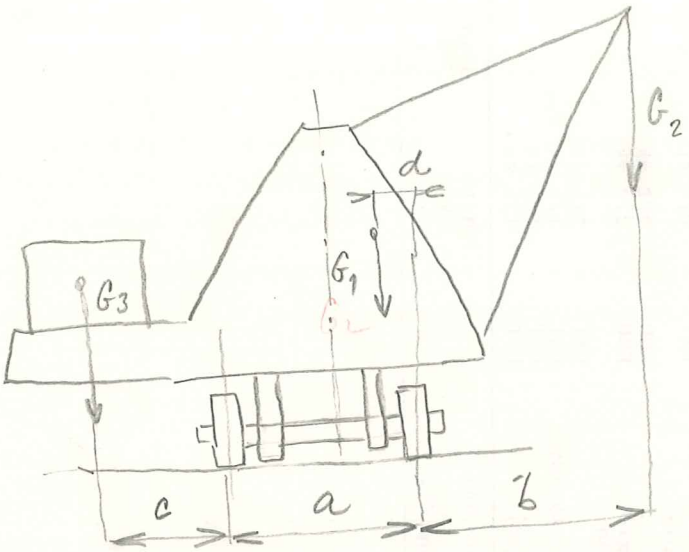
$x_1 = \frac{b}{2} = \frac{130}{2} = 65\ mm$
$x_2 = \frac{d}{2} = \frac{80}{2} = 40\ mm$
$x_3 = d + \frac{e}{3} = 80 + \frac{30}{3} = 90\ mm$

$S = S_1 - S_2 - S_3 = 7800 - 3200 - 600 = 4000\ mm^2$

$x_T \cdot S = S_1 \cdot x_1 - S_2 x_2 - S_3 x_3$

$$x_T = \frac{S_1 x_1 - S_2 x_2 - S_3 x_3}{S} = \frac{7800 \cdot 65 - 3200 \cdot 40 - 600 \cdot 90}{4000} = 81{,}25\ mm$$

1.103 $G_3 = ?$



$G_2 = 1,6\,MN$
$G_1 = 4,8\,MN$
$k = 1,8$
$a = 1,8\,m$
$b = 4,2\,m$
$c = 1,4\,m$
$d = 0,4\,m$

$$k = \frac{M_s}{M_k} \qquad M_s = G_1 d + G_3 (c+a) \qquad M_k = G_2 b$$

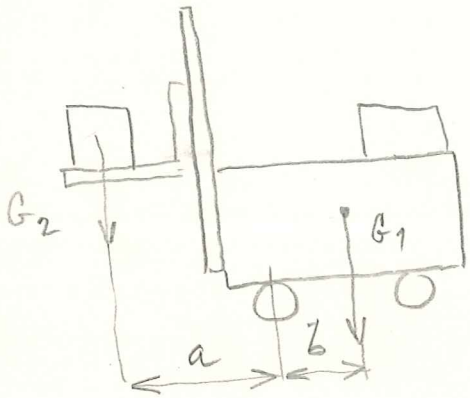
$$k M_k = M_s$$

$$k G_2 b = G_1 d + G_3 (c+a)$$

$$G_3 = \frac{k G_2 b - G_1 d}{c+a}$$

$$G_3 = \frac{1,8 \cdot 1,6 \cdot 4,2 - 4,8 \cdot 0,4}{1,4+1,8} = \underline{\underline{3,18\,MN}}$$

1.104



$G_2 = ?$
$G_1 = 6400\,N$
$k = 1,6$
$a = 2,1\,m$
$b = 1,7\,m$

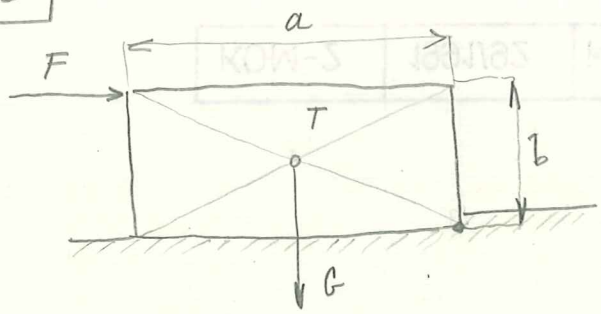
$M_s = G_1 b$
$M_k = G_2 a$

$$k = \frac{M_s}{M_k}$$

$$k = \frac{G_1 b}{G_2 a}$$

$$G_2 = \frac{G_1 b}{k a} = \frac{6400 \cdot 1,7}{1,6 \cdot 2,1} = \underline{\underline{3238\,N}}$$

1.102



$F = ?$

$k = 2{,}5$

$a = 820\ mm$

$b = 440\ mm$

$c = 650\ mm$

ocel $\varrho = 7800\ kg\,m^{-3}$

$G = m \cdot g \quad ; \quad m = V\varrho$

$V = abc = 0{,}82 \cdot 0{,}44 \cdot 0{,}65 = 0{,}2345\ m^3$

$m = V\varrho = 0{,}2345 \cdot 7800 = 1829{,}2\ kg$

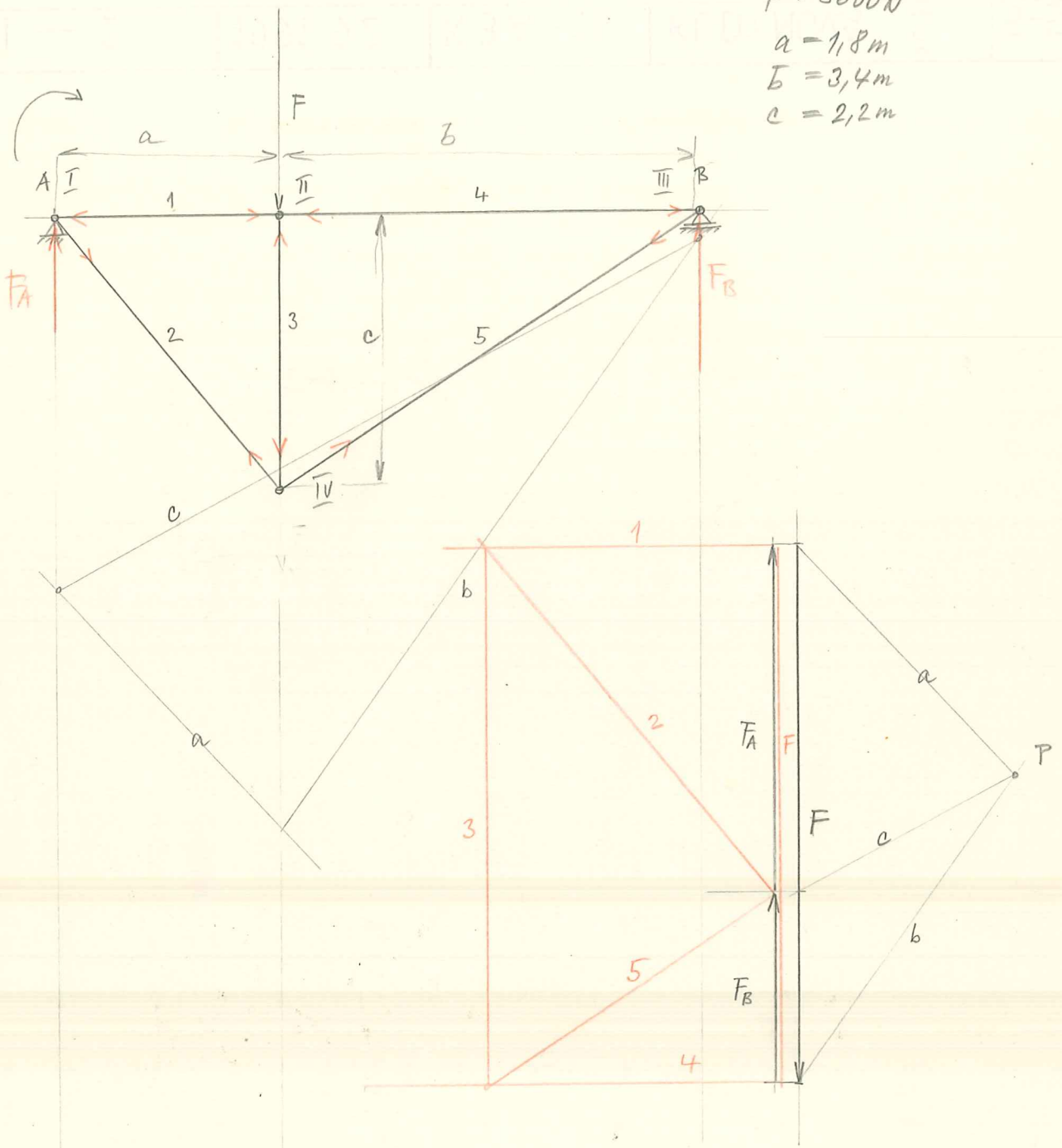
$G = m \cdot g = 1829{,}2 \cdot 9{,}81 = \underline{17945\ N}$

$$k = \frac{\Sigma M_s}{\Sigma M_c} = \frac{G\,\frac{a}{2}}{F \cdot b}$$

$$F = \frac{G \cdot a}{2k \cdot b} = \frac{17945 \cdot 0{,}82}{2 \cdot 2{,}5 \cdot 0{,}44} = \underline{6688{,}5\ N}$$

1.105

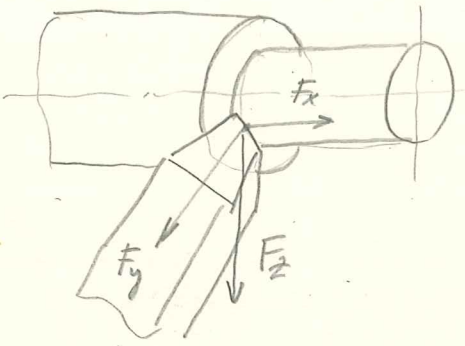
$F = 8600\,N$
$a = 1{,}8\,m$
$b = 3{,}4\,m$
$c = 2{,}2\,m$



$F_A = 5500\,N$
$F_B = 3100\,N$

$F_1 = -4600\,N$
$F_2 = +7200\,N$
$F_3 = -8600\,N$
$F_4 = -4600\,N$
$F_5 = +5500\,N$

1,116



$F_x = 240\,N$ $\quad F = ?$

$F_y = 650\,N$

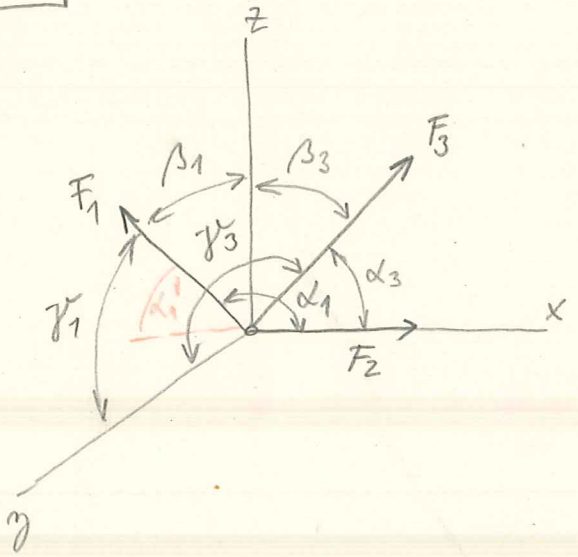
$F_z = 490\,N$

$$F = \sqrt{F_x^2 + F_y^2 + F_z^2} = \sqrt{240^2 + 650^2 + 490^2}$$

$$= \underline{\underline{848{,}6\,N}}$$

480 100

1,117



$F = ?$

$F_1 = F_2 = 3142\,N$

$F_3 = 1515\,N$

$\alpha_1 = 120°$ $\quad \gamma_1 = 77°$

$\alpha_2 = 0$ $\quad \gamma_2 = 90°$

$\alpha_3 = 45°$ $\quad \gamma_3 = 95°$

$\beta_1 = 60°$

$\beta_2 = 90°$

$\beta_3 = 45°$

Jaké směrové úhly svírá výslednice s osou x, y, z

F_1 ... $F_{1x} = F_1 \cos\alpha_1 = 3142 \cdot \cos 120° = -1571\,N$

$F_{1y} = F_1 \cos\gamma_1 = 3142 \cdot \cos 77° = 706{,}8\,N$

$F_{1z} = F_1 \cos\beta_1 = 3142 \cdot \cos 60° = 1571\,N$

F_2 ... $F_{2x} = F_2 = 3142\,N$

$F_{2y} = 0$

$F_{2z} = 0$

F_3 ... $F_{3x} = F_3 \cos\alpha_3 = 1515 \cdot \cos 45° = 1071{,}2\,N$

$F_{3y} = F_3 \cos\gamma_3 = 1515 \cdot \cos 95° = -132\,N$

$F_{3z} = F_3 \cos\beta_3 = 1515 \cdot \cos 45° = 1071{,}2\,N$

$$F_x = F_{1x} + F_{2x} + F_{3x} = -1571 + 3142 + 1071{,}2 = 2642{,}2\,N$$

$$F_y = F_{1y} + F_{2y} + F_{3y} = 706{,}8 + 0 - 132 = 574{,}8\,N$$

$$F_z = F_{1z} + F_{2z} + F_{3z} = 1571 + 0 + 1071{,}2 = 2642{,}2\,N$$

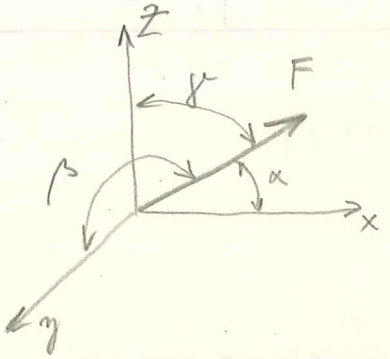
$$F = \sqrt{F_x^2 + F_y^2 + F_z^2} = \sqrt{2642{,}2^2 + 574{,}8^2 + 2642{,}2^2} = \underline{3780{,}5\,N}$$

$\cos\alpha = \frac{F_x}{F} = \frac{2642{,}2}{3780{,}5} = 0{,}698$ $\quad \underline{\alpha = 45{,}66°}$

$\cos\gamma = \frac{F_y}{F} = \frac{574{,}8}{3780{,}5} = 0{,}152$ $\quad \underline{\gamma = 81{,}25°}$

$\cos\beta = \frac{F_z}{F} = \frac{2642{,}2}{3780{,}5} = 0{,}698$ $\quad \underline{\beta = 45{,}66°}$

1.118



$F = 510\,N$

$F_x = ?$ $\quad \alpha = 60°$

$F_y = ?$ $\quad \beta = 50°$

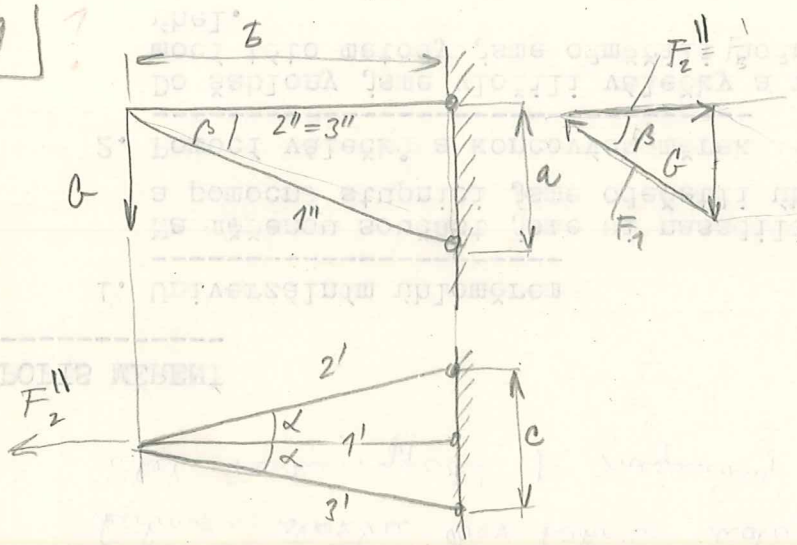
$F_z = ?$ $\quad \gamma = 54°34'$

$F_x = F\cos\alpha = 510 \cdot \cos 60° = 255\,N$

$F_y = F\cos\beta = 510 \cdot \cos 50° = 327{,}8\,N$

$F_z = F\cos\gamma = 510 \cdot \cos 54{,}5° = 296{,}15\,N$

1.119



$G = 2400\,N$

$a = 1{,}5\,m$

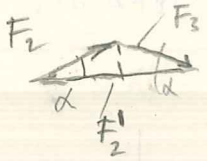
$b = 0{,}875\,m$

$c = 0{,}9\,m$

$tg\beta = \frac{a}{b} = \frac{1{,}5}{0{,}875} = 1{,}714 \dots \beta = 59{,}74°$

$F_1 = \frac{G}{\sin\beta} = \frac{2400}{0{,}863} = 2778{,}5\,N$

$F_2'' = \frac{G}{tg\beta} = \frac{2400}{tg\,59{,}74°} = 1400{,}2\,N$



$F_2' = \frac{F_2''}{2\cdot\cos\alpha} = \frac{1400{,}2}{2\cdot\cos 27{,}22} = 787{,}2\,N$

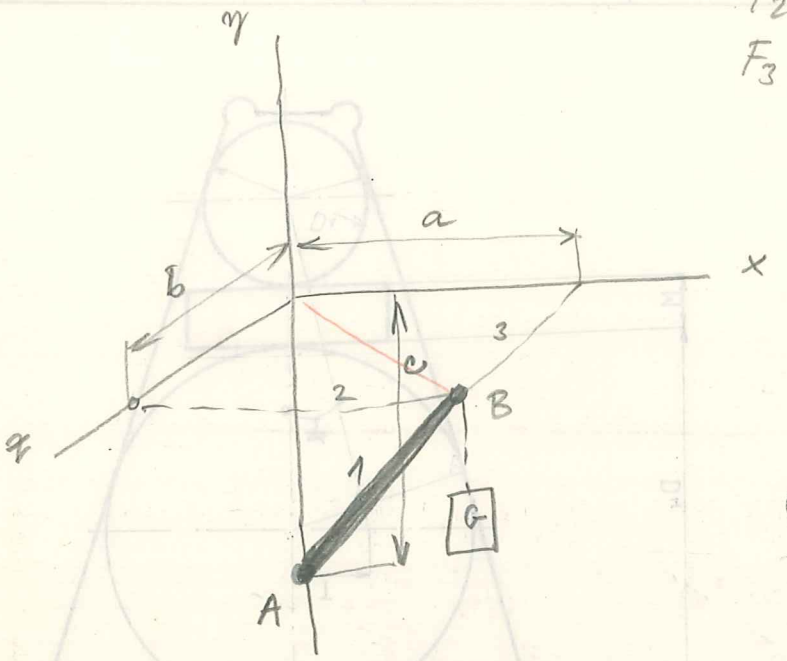
$tg\alpha = \frac{\frac{c}{2}}{b} = \frac{0{,}9}{2\cdot 0{,}875} = 0{,}5142$

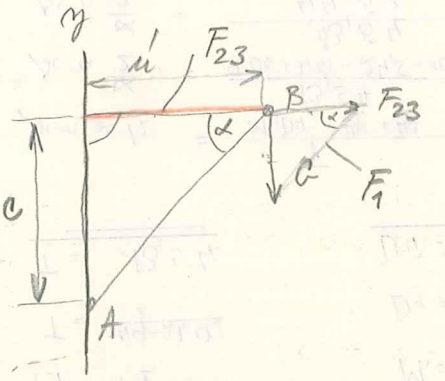
$\alpha = 27{,}22°$

1.120

$F_1 = ?$
$F_2 = ?$
$F_3 = ?$

$G = 5200\ N$
$a = 3{,}8\ m$
$b = 3{,}2\ m$
$c = 2{,}7\ m$



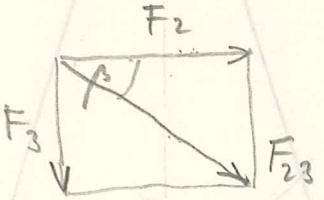


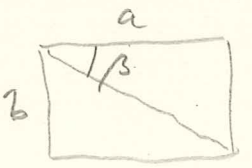
$$u' = \sqrt{a^2 + b^2} = \sqrt{3{,}8^2 + 3{,}2^2} = 4{,}97\ m$$

$$tg\,\alpha = \frac{c}{u'} = \frac{2{,}7}{4{,}97} = 0{,}5432 \quad \underline{\alpha = 28{,}57^\circ}$$

$$F_1 = \frac{G}{\sin\alpha} = \frac{5200}{\sin 28{,}57^\circ} = \underline{10\,894\ N}$$

$$\frac{G}{F_{23}} = tg\,\alpha \rightarrow F_{23} = \frac{G}{tg\,\alpha} = \frac{5200}{0{,}5432} = \underline{9572{,}9\ N}$$





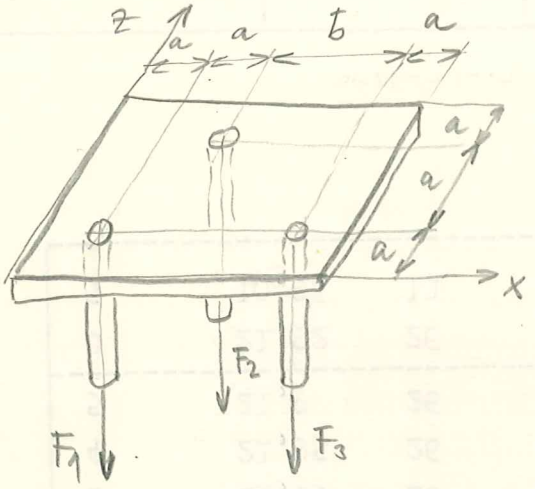
$$\frac{b}{a} = tg\,\beta = \frac{3{,}2}{3{,}8} = 0{,}842$$

$$\underline{\beta = 40{,}1^\circ}$$

$$F_2 = F_{23}\cos\beta = 9572{,}9 \cdot \cos 40{,}1^\circ = \underline{7322{,}5\ N}$$

$$F_3 = F_{23}\sin\beta = 9572{,}9 \cdot \sin 40{,}1^\circ = \underline{6166{,}1\ N}$$

1,121



$F = ?$ $\quad x = ?$ $\quad z = ?$

$F_1 = 500\,N$

$F_2 = 3000\,N$

$F_3 = 1500\,N$

$a = 20\,mm$

$b = 80\,mm$

$$F = F_1 + F_2 + F_3 = 500 + 3000 + 1500 = 5000\,N$$

$$M_x: \quad F_1 \cdot a + F_2 \cdot 2a + F_3 \cdot a = F \cdot z$$

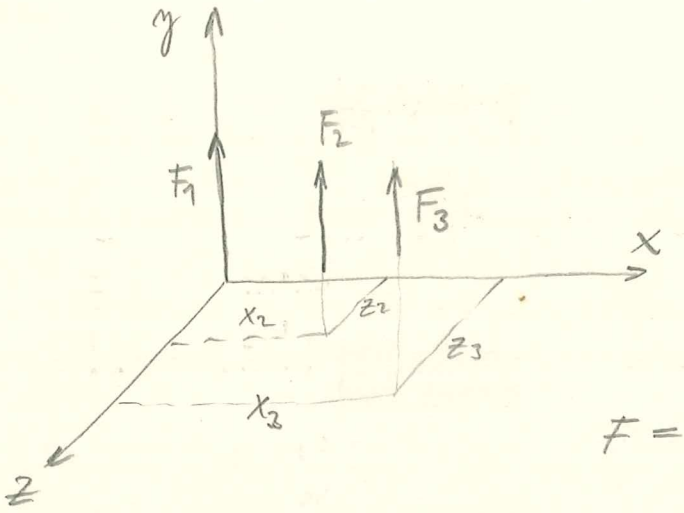
$$z = \frac{a(F_1 + 2F_2 + F_3)}{F} = \frac{20(500 + 6000 + 1500)}{5000} = 32\,mm$$

$$M_z: \quad F_1 \cdot a + F_2 \cdot 2a + F_3(2a + b) = F \cdot x$$

$$x = \frac{500 \cdot 20 + 3000 \cdot 40 + 1500(40 + 80)}{5000} = \frac{10000 + 120000 + 180000}{5000}$$

$$= 47\,mm$$

1.122



$F_1 = 400\ N$
$F_2 = 800\ N$
$F_3 = 1000\ N$
$x_2 = 20\ mm$
$x_3 = 60\ mm$
$z_2 = 40\ mm$
$z_3 = 10\ mm$

$F = F_1 + F_2 + F_3 = 400 + 800 + 1000 = \underline{2200\ N}$

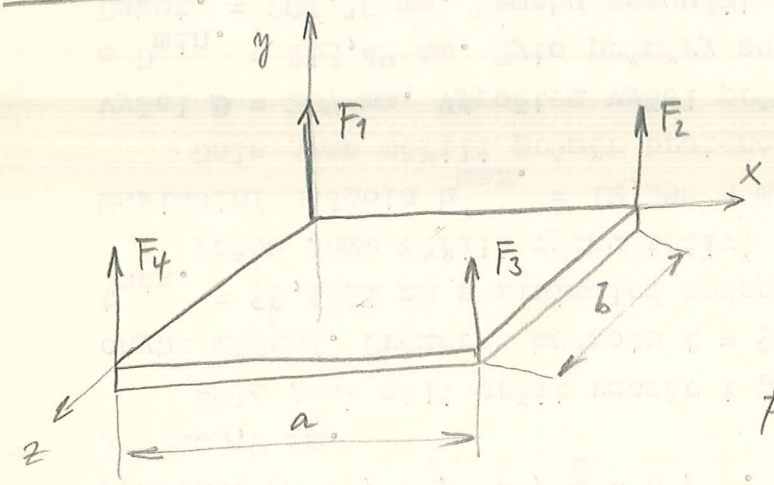
$M_x \dots \quad F_2 \cdot z_2 + F_3 \cdot z_3 = F \cdot z$

$$z = \frac{F_2 z_2 + F_3 z_3}{F} = \frac{800 \cdot 40 + 1000 \cdot 10}{2200} = \underline{19{,}1\ mm}$$

$M_z \dots \quad F_2 x_2 + F_3 x_3 = F \cdot x$

$$x = \frac{F_2 x_2 + F_3 x_3}{F} = \frac{800 \cdot 20 + 1000 \cdot 60}{2200} = \underline{34{,}54\ mm}$$

1.123



$F_1 = 500\ N$
$F_2 = 600\ N$
$F_3 = 100\ N$
$F_4 = 300\ N$
$a = 800\ mm$
$b = 600\ mm$

$F = F_1 + F_2 + F_3 + F_4$
$= 500 + 600 + 100 + 300 = \underline{1500\ N}$

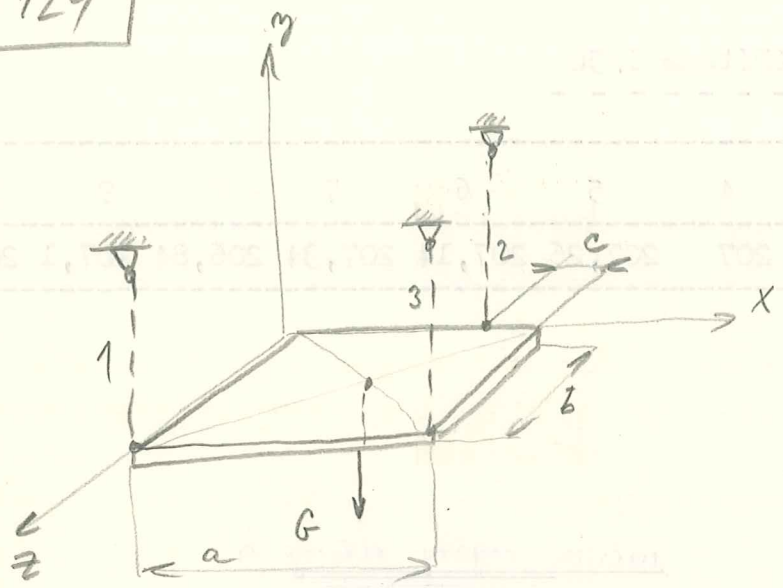
$Mx \dots \quad F_4 \cdot b + F_3 \cdot b = F \cdot z$

$$z = \frac{b(F_4 + F_3)}{F} = \frac{600\,(300 + 100)}{1500} = \underline{160\ mm}$$

$M_z \dots \quad F_3 \cdot a + F_2 \cdot a = F \cdot x$

$$x = \frac{a(F_3 + F_2)}{F} = \frac{800\,(100 + 600)}{1500} = \underline{373{,}\overline{3}\ mm}$$

1.124



$G = 1000\ N$

$F_1 = ?$

$F_2 = ?$

$F_3 = ?$

$a = 1\ m$

$b = 0{,}6\ m$

$c = 0{,}3\ m$

$$F_1 + F_2 + F_3 - G = 0$$

$M_z \ldots \quad F_3 \cdot a + F_2(a-c) - G\frac{a}{2} = 0$

$M_x \ldots \quad F_1 b + F_3 b - G\frac{b}{2} = 0$

$$F_3 = \frac{G\frac{b}{2} - F_1 b}{b} = \frac{G}{2} - F_1$$

$$F_2 = \frac{G\frac{a}{2} - F_3 \cdot a}{a-c} = \frac{G\frac{a}{2} - G\frac{a}{2} + F_1 a}{a-c} = \frac{F_1 a}{a-c}$$

$$F_1 + \frac{F_1 a}{a-c} + \frac{G}{2} - F_1 - G = 0$$

$$F_1 \cdot a + \frac{G}{2}(a-c) - G(a-c) = 0$$

$$F_1 = \frac{(a-c)\left(G - \frac{G}{2}\right)}{a} = \frac{a-c}{2a} G = \frac{1-0{,}3}{2 \cdot 1} \cdot 1000 = 350\ N$$

$$F_2 = \frac{F_1 a}{a-c} = \frac{350 \cdot 1}{1-0{,}3} = 500\ N$$

$$F_3 = G - F_1 - F_2 = 1000 - 350 - 500 = 150\ N$$

MEC 1

A 1.125 $G = 340\,N$ dřevo – dřevo

Graficky a početně $f_0 = 0{,}5$

$f_0 = 0{,}16$

a) suché

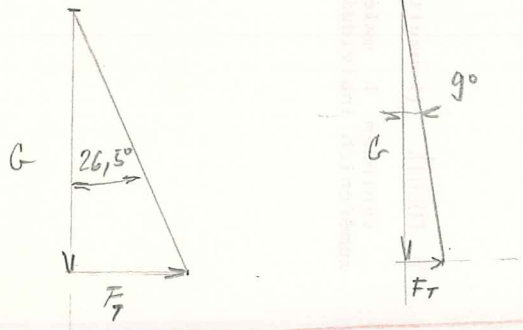
b) mazané

a) $F = f_0 G = 0{,}5 \cdot 340 = 170\,N$

b) $F = f_0 G = 0{,}16 \cdot 340 = 54{,}4\,N$

a) graf. $tg\,\varphi_0 = f_0 = 0{,}5 \ldots\ldots \varphi_0 = 26{,}5°$

b) $tg\,\varphi_0 = f_0 = 0{,}16 \quad \varphi_0 = 9{,}09°$



B 1.126 $f_0 = ?$ dřevo – dřevo

$F = 716\,N$

$2240 \times 760 \times 250$ $\quad \rho = 840\,kg \cdot m^{-3}$

$V = 2{,}24 \cdot 0{,}76 \cdot 0{,}25 = 0{,}4256\,m^3$

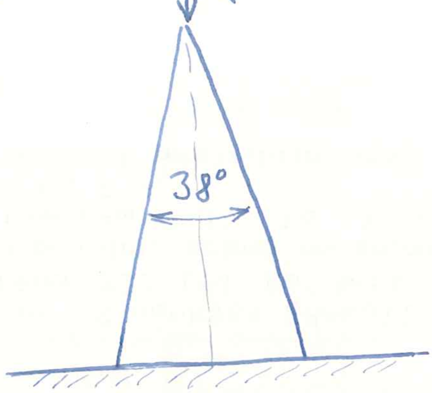
$m = V \cdot \rho = 0{,}4256 \cdot 840 = 357{,}5\,kg$

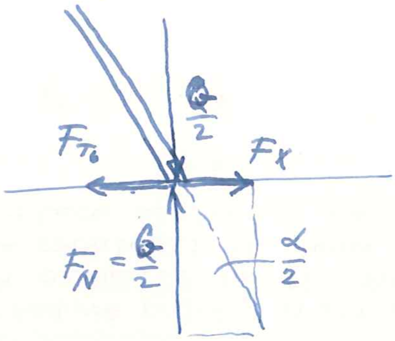
$G = mg = 357{,}5 \cdot 9{,}81 = 3507\,N$

$f_0 = \frac{F}{G} = \frac{716}{3507} = \underline{\underline{0{,}204}}$

MEC 1

1.127 $Q = 746\,N \; \dots \; f_0 = ?$ aby se žebřík nerozjel





$F_x = F_{T_0}$

$F_x = F_N \operatorname{tg} \frac{\alpha}{2}$

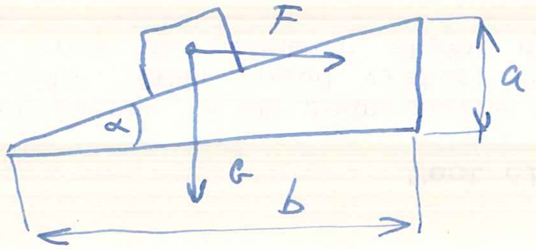
$F_{T_0} = F_N \operatorname{tg} \frac{\alpha}{2}$

$F_N \cdot f_0 = F_N \operatorname{tg} \frac{\alpha}{2}$

$f_0 = \operatorname{tg} \frac{\alpha}{2} \quad \dots \quad f_0 = \operatorname{tg} \frac{38}{2} = 0{,}344$

1.128 Stejný typ příkladu probrán ve výuce

1.129 Pohyb tělesa pohybem vzhůru



$F = ?$

$G = 840\,N$

$f = 0{,}13$

$a = 36$

$b = 870$

$\operatorname{tg} \alpha = \frac{a}{b} = \frac{36}{870} = 0{,}04137 \quad \dots \quad \alpha = 2{,}3689°$

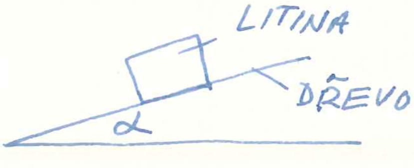
$\operatorname{tg} \varphi = 0{,}13 \quad \dots \quad \varphi = 7{,}4069$ (~~7,4~~) ?

~~[illegible]~~ 9,77

$F = G \operatorname{tg}(\alpha + \varphi) = 840 \operatorname{tg} 9{,}77 = \underline{144{,}5\,N}$

= ~~[illegible]~~

1.130



$\alpha = 26°5'$

$f = ?$ mezi litinou a dřevem

$\operatorname{tg} \alpha = f = 0{,}489$