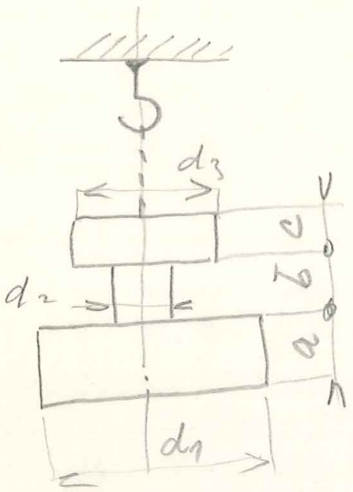
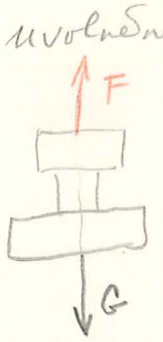
STATIKA

1,3 Vypočtěte sílu na lávku na kterém je zavěšena ocelová soudobá kruhové prářezem obr.

$d_1 = 250\,mm$ $a = 60\,mm$
$d_2 = 120\,mm$ $b = 140\,mm$
$d_3 = 180\,mm$ $c = 80\,mm$





$\rho = 7850\,kg.m^{-3}$

$V = V_1 + V_2 + V_3 = \frac{\pi d_1^2}{4}.a + \frac{\pi d_2^2}{4}.b + \frac{\pi d_3^2}{4}.c =$

$= \frac{\pi}{4}\left(d_1^2 a + d_2^2.b + d_3^2.c\right) =$

$= \frac{\pi}{4}\left(0{,}25^2.0{,}06 + 0{,}12^2.0{,}14 + 0{,}18^2.0{,}08\right) =$

$= \frac{\pi}{4}\left(3{,}75.10^{-3} + 2{,}016.10^{-3} + 2{,}592.10^{-3}\right) =$

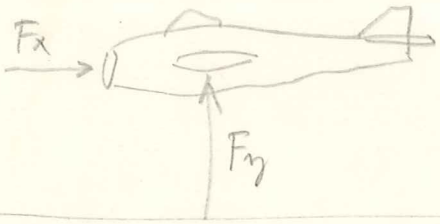
$= 6{,}564.10^{-3}\,m^3$

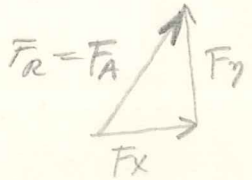
$m = \rho V = 7850.6{,}564.10^{-3} =$
$\doteq 51{,}53\,kg$

$G = m.g = 51{,}53.9{,}81 = \underline{505{,}5\,N}$

$F = |G| = 505{,}5\,N$

1.4 Na letadlo působí $F_x = 1350\,N$ odpor vzduchu a vztlaková síla $F_y = 4800\,N$. Jak je velká $F_A = ?$ aerodynamická síla jako síla působená z obrázku





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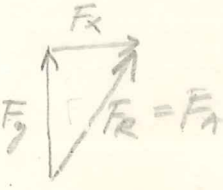
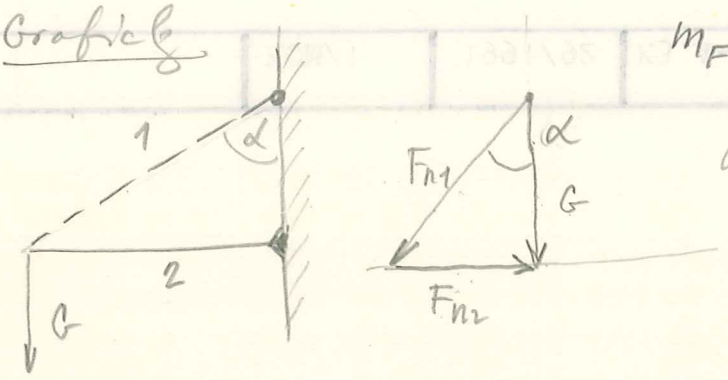


$F_A = \sqrt{F_x^2 + F_y^2} = \sqrt{1350^2 + 4800^2} = \underline{4986{,}2\,N}$

1.5 Podobně a grafické určete F_{N1} v tyči 1 a sílu F_{N2} v prutu 2. $G = 690\,N$, $\alpha = 69°$

a) Grafické

m_F ---- 1 mm = 10 N



Odměřené F_{n1} =

F_{n2} =

b) Početně

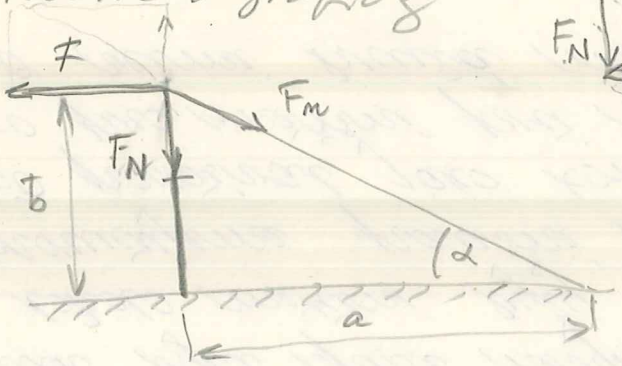
$$\frac{G}{F_{n1}} = \cos\alpha$$

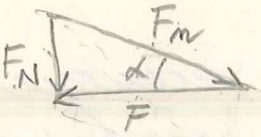
$$F_{n1} = \frac{G}{\cos\alpha^\circ} = \frac{690}{\cos 69^\circ} = \underline{1925{,}4\,N}$$

$$F_{n2} = G \cdot \operatorname{tg}\alpha = 690 \cdot \operatorname{tg} 69^\circ = \underline{1797{,}5\,N} \qquad \frac{F_{n2}}{G} = \operatorname{tg}\alpha$$

1.6 Zjistěte sílu F_n v kolmém směru, kterou je nutno působit kamenem a F_N, kterou je tlačen působící k zemi, když síla síla v lanu $F = 24\,300\,N$; $a = 17\,m$, $b = 12\,m$.

Početně v grafické





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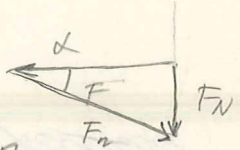


$$\operatorname{tg}\alpha = \frac{b}{a} = \frac{12}{17} = 0{,}7058$$

$$\underline{\alpha = 35{,}22^\circ}$$

$$F_N = F \operatorname{tg}\alpha = \qquad \frac{F_N}{F} = \operatorname{tg}\alpha$$

$$= 24\,300 \cdot 0{,}7058 = \underline{17\,150\,N}$$

$$F_n = \frac{F}{\cos\alpha} = \qquad \frac{F}{F_n} = \cos\alpha$$

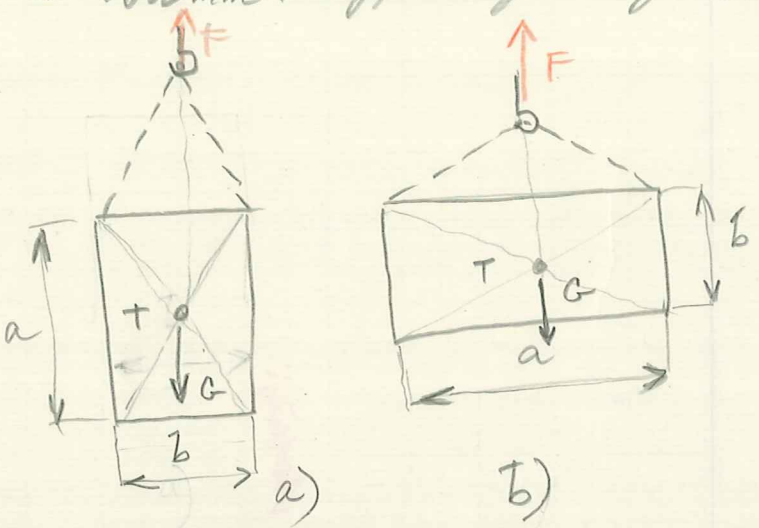
$$= \frac{24\,300}{\cos 35{,}22^\circ} = \underline{29\,745\,N}$$

1.7 Ne

1.8 Ocelová deska o rozměrech $a = 1600\,mm$, $b = 1230\,mm$ a tloušťce $t = 14\,mm$ zavěšená jedním bodem pomocí lana o délce $l = 1900\,mm$. Vypočtěte síly v lanech a); b)

$\rho = 7850\,kg \cdot m^{-3}$

$F = |G|$



$G = mg$

$m = V\rho = abt\rho =$

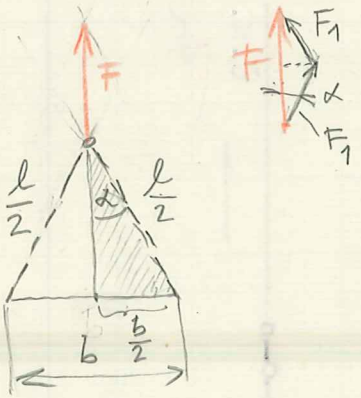
$= 1{,}6 \cdot 1{,}23 \cdot 0{,}014 \cdot 7850 =$

$\doteq 216{,}3\,kg$

$G = mg = 216{,}3 \cdot 9{,}81 = 2121{,}9$

$= 2122\,N$

a)



$$\sin\alpha_1 = \frac{\frac{b}{2}}{\frac{l}{2}} = \frac{b}{l} = \frac{1230}{1900} = 0{,}674$$

$$\alpha_1 = 40{,}34°$$

$F = G$

$$\cos\alpha = \frac{\frac{F}{2}}{F_1} \Rightarrow F_1 = \frac{F}{2\cos\alpha}$$

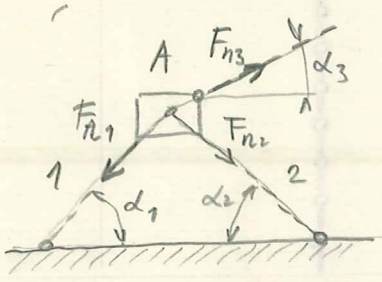
$$F_1 = \frac{2122}{2\cos 40{,}34} = 1391{,}9\,N$$

---

b) $\sin\alpha_2 = \frac{a}{l} = \frac{1600}{1900} = 0{,}842$ ---- $\alpha_2 = 57{,}36°$

$$F_1 = \frac{F}{2\cos\alpha_2} = \frac{2122}{2\cos 57{,}36} = 1967{,}1\,N$$

1.9 Těleso A je upevněno třemi lany podle obr. Jaká síla F_{n3} musí působit v lanu 3, když síly $F_{n1} = 93\,N$, $F_{n2} = 62\,N$ v lanech 1, 2. Těleso A je nevážné velikosti. $\alpha_1 = 43°$, $\alpha_2 = 52°$. Jak velká musí být úhel α_3, aby lano 3 trhat nemohlo?



Rovnováha

$F_{n1x} = F_{n1} \cos\alpha_1 = 93 \cos 43° = 68\,N$

$F_{n1y} = F_{n1} \sin\alpha_1 = 93 \sin 43° = 63,4\,N$

$F_{n2x} = F_{n2} \cos\alpha_2 = 62 \cos 52° = 38,2\,N$

$F_{n2y} = F_{n2} \sin\alpha_2 = 62 \sin 52° = 48,8\,N$

$F_{n3x} = ?$

$F_{n3y} = ?$

$\Sigma F_x = 0 \quad \ldots \quad -F_{n1x} + F_{n2x} + F_{n3x} = 0$

$F_{n3x} = F_{n1x} - F_{n2x} = 68 - 38,2 = \underline{29,8\,N}$

$\Sigma F_y = \quad \ldots \quad -F_{n1y} - F_{n2y} + F_{n3y} = 0$

$F_{n3y} = F_{n1y} + F_{n2y} = 63,4 + 48,8 = \underline{112,2\,N}$

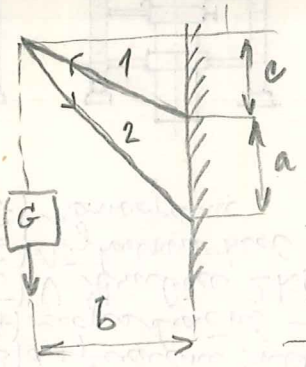
$F_{n3} = \sqrt{F_{n3x}^2 + F_{n3y}^2} = \sqrt{29,8^2 + 112,2^2} = \underline{116,1\,N}$

$tg\,\alpha_3 = \frac{F_{n3y}}{F_{n3x}} = \frac{112,2}{29,8} = 3,765$

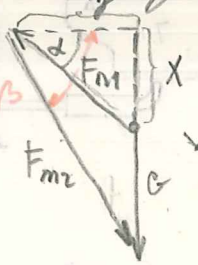
$\underline{\alpha_3 = 75,12°}$

1.10) Na nástěnné konzole je břemeno $G = 340\,N$, uvnitř úhly α, β F_{n1}, F_{n2} v prutech 1, 2. Rozměry $a = 1,4\,m$; $b = 2,3\,m$; $c = 1,1\,m$

1400 2300 1100

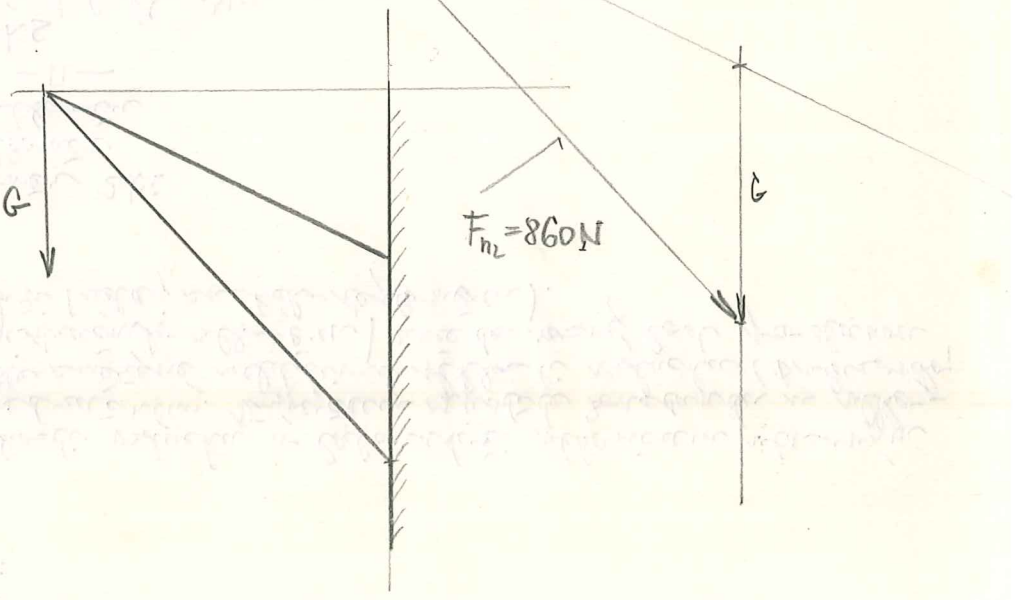


G – výslednice

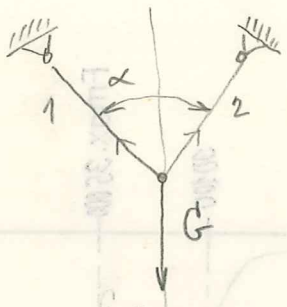


Grafický

$F_{n1} = 65.10 = 650\,N$



1.11

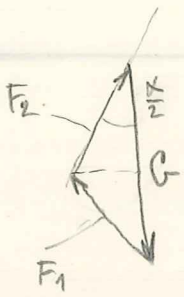


$G = 720\,N$

$\alpha = 80°$

$F_1 = ?$

$F_2 = ?$

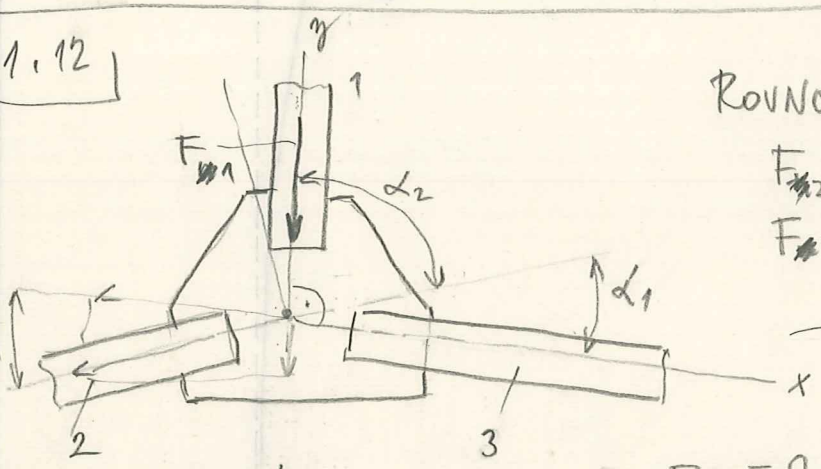


$$\frac{\frac{G}{2}}{F_2} = \cos\frac{\alpha}{2}$$

$$F_2 = \frac{G}{2\cos\frac{\alpha}{2}} = \frac{720}{2\cos 40°} = \underline{469{,}9\,N}$$

$F_1 = F_2$

1.12



Rovnováha

$F_{n2} = ?$ $\quad F_{n1} = -1320\,N$

$F_{n3} = ?$ $\quad \alpha_1 = 30°$

$\alpha_2 = 60°$

Postup

$F_{1x} = 0$

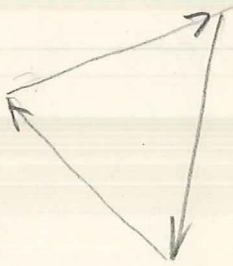
$F_{1y} = -F_{n1}$

$F_{2x} = F_{n2}\cos\alpha_1$

$F_{2y} = F_{n2}\sin\alpha_1$

$F_{3x} = F_{n3}$

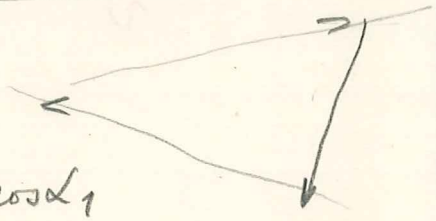
$F_{3y} = 0$



$\Sigma F_x = 0$

$0 - F_{2x} + F_{3x} = 0$

$F_{n3} = F_{n2}\cos\alpha_1$



$\Sigma F_y = 0$

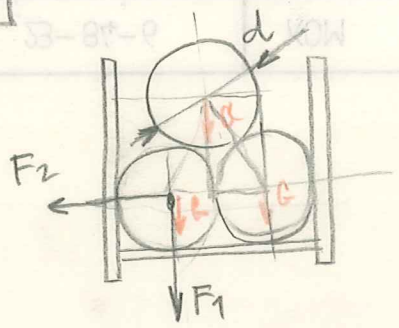
$-F_{n1} - F_{2y} + 0 = 0$

$F_{n1} = -F_{n2}\sin\alpha_1$

$$F_{n2} = -\frac{-F_{n1}}{\sin\alpha_1} = -\frac{-1320}{\sin 30°} = \underline{+2640\,N}$$

$$F_{n3} = +2640\cos 30° = \underline{+2286{,}3\,N}$$

1.13

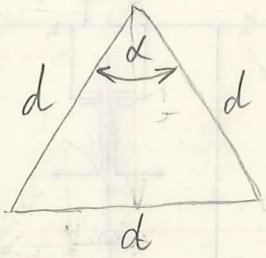


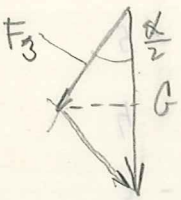
$d = 180$ a $l = 2560$ mm

$G = 2300$ N

Jakou silou tlačí spodní trubky na zem a na bočnice?

$\alpha = 60°$

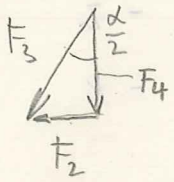




$$F_3 = \frac{\frac{G}{2}}{\cos\frac{\alpha}{2}} =$$

$$F_2 = F_3 \sin\frac{\alpha}{2} = \frac{G}{2\cos\frac{\alpha}{2}} \sin\frac{\alpha}{2}$$

$$= \frac{G}{2} \operatorname{tg}\frac{\alpha}{2} = \frac{2300}{2} \operatorname{tg} 30 = 663,$$



$$F_4 = F_3 \cdot \cos\frac{\alpha}{2} = \frac{G}{2\cos\frac{\alpha}{2}} \cos\frac{\alpha}{2} =$$

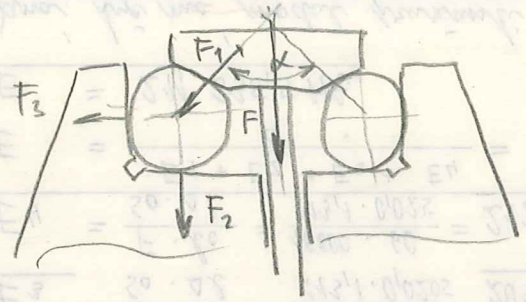
$$= \frac{G}{2} = \frac{2300}{2} =$$

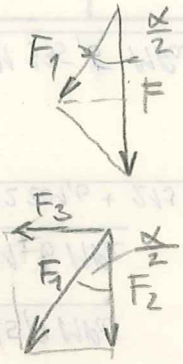
$$F_1 = G + F_4 = 2300 + \frac{2300}{2} =$$

$$= 3450 \text{ N}$$

$$F_2 = 664 \text{ N}$$

1.14





$F = 465\,N$ $F_2 = ?$
$\alpha = 50°$ $F_3 = ?$

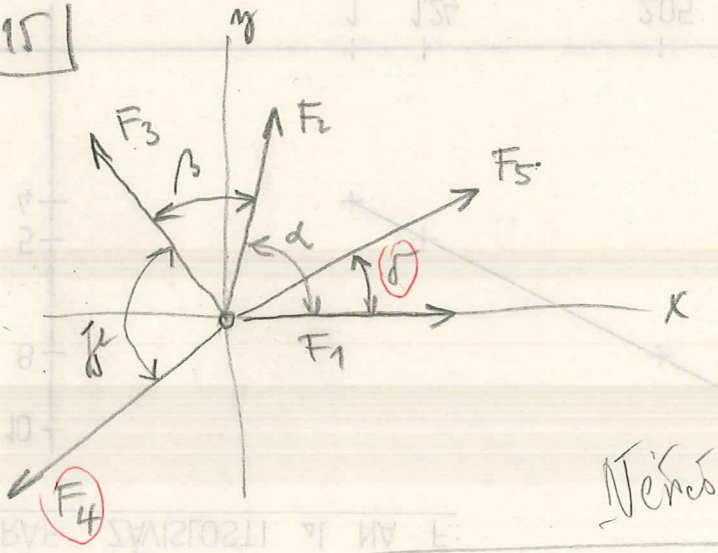
$$\frac{\frac{F}{2}}{F_1} = \cos\frac{\alpha}{2}$$

$$F_1 = \frac{F}{2\cos\frac{\alpha}{2}}$$

$$F_2 = F_1\cos\frac{\alpha}{2} = \frac{F}{2\cos\frac{\alpha}{2}}\cos\frac{\alpha}{2} = \frac{F}{2} = \frac{465}{2} = 232{,}5\,N$$

$$F_3 = F_1\sin\frac{\alpha}{2} = \frac{F}{2}\frac{\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} = \frac{F}{2}\operatorname{tg}\frac{\alpha}{2} = 108{,}4\,N$$

1.15



$F_1 = 710\,N$ $\alpha = 35°$
$F_2 = 540\,N$ $\beta = 71°$
$F_3 = 660\,N$ $\gamma = 112°$
$F_5 = 536\,N$ $\delta = ?$

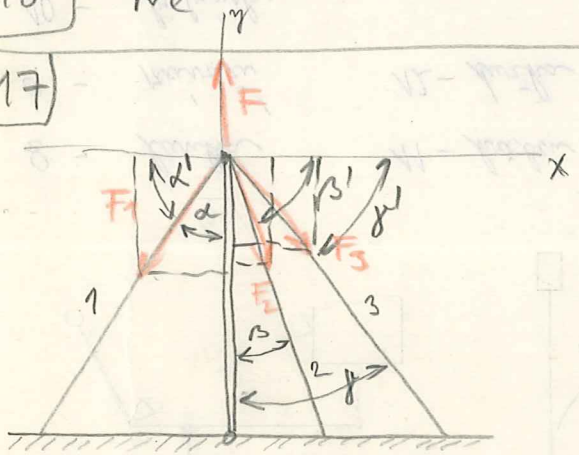
Rovnováha

Nerozhodne

1.16

Ne

1.17



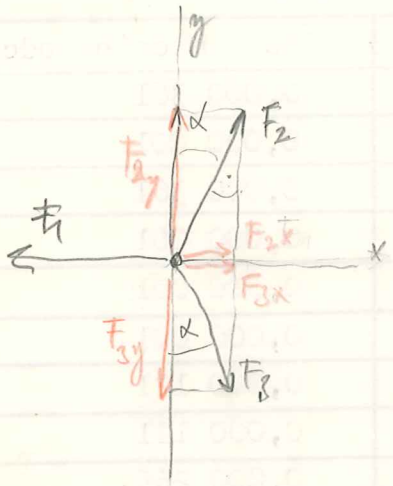
$F_1 = 518\,N$ $\alpha = 34°$ $\alpha' = 56°$
$F_2 = 287\,N$ $\beta = 23°$ $\beta' = 67°$
$F_3 = 329\,N$ $\gamma = ?$

$F = ?$

$F_{1x} = F_1\cos\alpha' = 518\cos 56° = 289{,}6\,N$
$F_{1y} = F_1\sin\alpha' = 518\sin 56° = 429{,}4\,N$
$F_{2x} = F_2\cos\beta' = 287\cos 67° = 112{,}1\,N$
$F_{2y} = F_2\sin\beta' = 287\sin 67° = 264{,}1\,N$
$F_{3x} = F_3\cos\gamma' = 329\cos\gamma'$
$F_{3y} = F_3\sin\gamma' = 329\sin\gamma'$

1.18

$F_1 = 97500\,N$, $F_2 = F_3 = 41000\,N$, $\alpha = 45°$



$F_{2x} = F_2 \sin\alpha = 41000 \cdot \sin 45° = 28991\,N$

$F_{2y} = F_2 \cos\alpha = 41000 \cos 45° = 28991\,N$

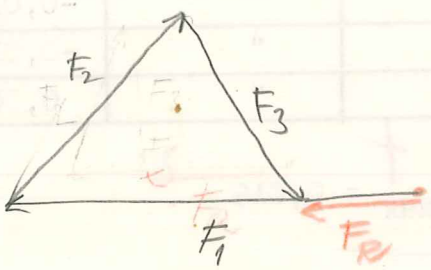
$F_{3x} = F_{2x} = 28991\,N$

$F_{3y} = F_{2y} = 28991\,N$

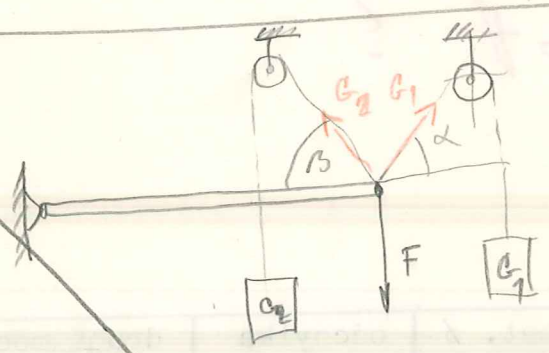
$F_{Rx} = -F_1 + F_{2x} + F_{3x} = -97500 + 2 \cdot 28991 = -39517{,}2\,N$

$F_{Ry} = +F_{2y} - F_{3y} = 0$

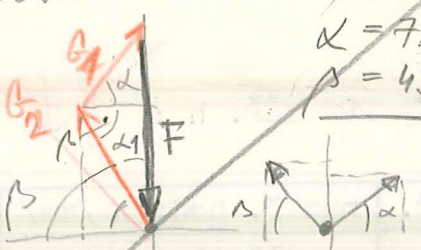
Graficky



1.19



Rovnováha

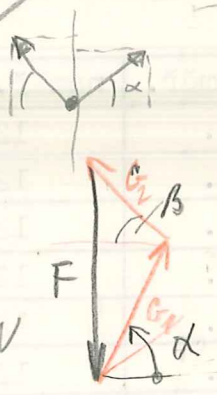


$G_1 = 9600\,N$

$G_2 = 7350\,N$

$\alpha = 72°$

$\beta = 43°$



$G_{1x} = G_1 \cos\beta = 9600 \cdot \cos 43° = 7020\,N$

$G_{1y} = G_1 \sin\beta = 9600 \cdot \sin 43° = 6547\,N$

$G_{2x} = G_2 \cos\alpha = 7350 \cos 72° = 2271{,}2\,N$

$G_{2y} = G_2 \sin\alpha = 7350 \sin 72° = 6990\,N$

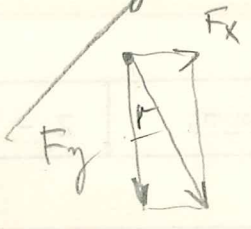
$\Sigma F_x = 0$

$F_x - G_{1x} + G_{2x} = 0$

$F_x = G_{1x} - G_{2x} = 7020 - 2271{,}2 = 4749$

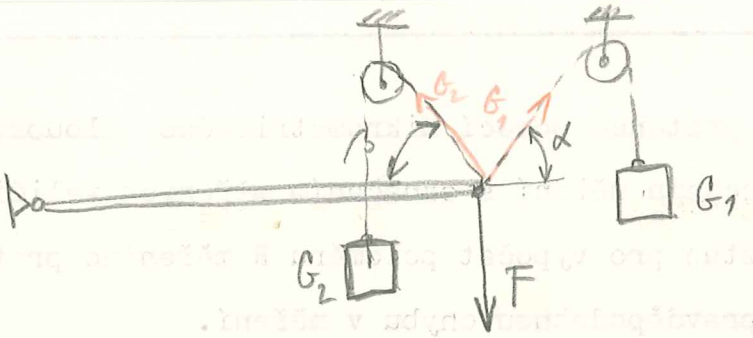
$\Sigma F_y = 0 \quad -F_y + G_{1y} + G_{2y} = 0$

$F_y = G_{1y} + G_{2y} = 6547 + 6990 = 13537$



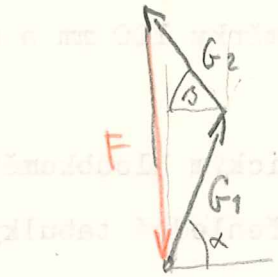
$\tan\gamma = \frac{F_y}{F_x} = 70°$

1.19

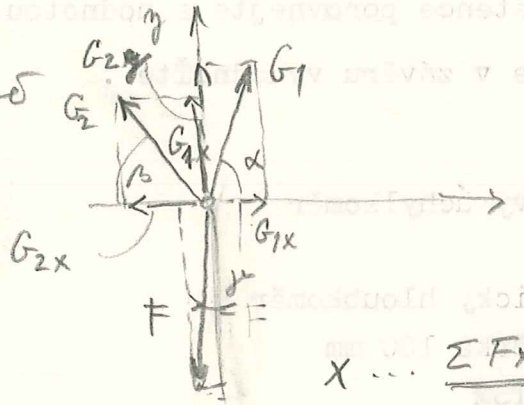


$G_1 = 9600\,N$
$G_2 = 7350\,N$
$\alpha = 72°$
$\beta = 43°$

Grafické řešení



Početní řešení



$G_{1x} = G_1 \cos\alpha = 9600 \cos 72° = 2966{,}5\,N$
$G_{1y} = G_1 \sin\alpha = 9600 \sin 72° = 9130\,N$
$G_{2x} = G_2 \cos\beta = 7350 \cdot \cos 43° = 5375\,N$
$G_{2y} = G_2 \sin\beta = 7350 \cdot \sin 43° = 5012{,}6\,N$

$x \dots \Sigma F_x = 0$

$-F_x + G_{1x} - G_{2x} = 0$

$-F_x = G_{2x} - G_{1x} = 5375 - 2966{,}5$

$= 2408{,}5\,N$

$y \dots \Sigma F_y = 0$

$-F_y + G_{1y} + G_{2y} = 0$

$F_y = G_{1y} + G_{2y} = 9130 + 5012{,}6 =$

$= 14142{,}6\,N$

$\operatorname{tg}\gamma = \frac{F_x}{F_y} = \frac{2408{,}5}{14142{,}6} = 0{,}1703$

$\gamma = 9{,}66°$

$F = \sqrt{F_x^2 + F_y^2} = \sqrt{2408{,}5^2 + 14142{,}6^2} = 14346\,N$

678 kN

1.22 Určete moment dvojice sil $F = 580\,N$ když rameno $a = 460\,mm$. $M\,[Nmm]$ a $[Nm]$

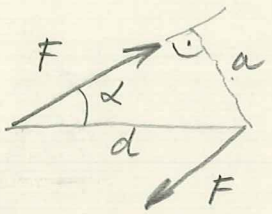
$$M = F.a = 580.460 = \underline{266\,800\,Nmm} = \underline{266,8\,Nm}$$

1.23 Dvojice sil $M = 1280\,Nm$ rameno je $a = 250\,mm$; $F = ?$

$$F = \frac{M}{a} = \frac{1280}{0,25} = \underline{5120\,N}$$

1.24 Určete moment dvojice sil $F = 422\,N$; $d = 160\,mm$; $\alpha = 27°$

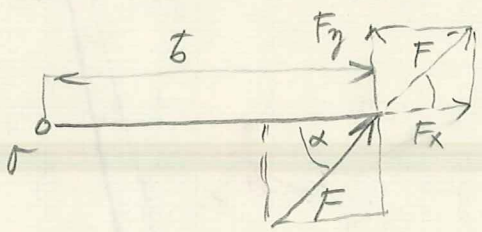
$M = ?\,[Nmm]$



$$a = d.\sin\alpha = 160.\sin 27° = 72,64\,mm$$

$$M = F.a = 422.72,64 = \underline{30654\,Nmm}$$

1.25 Určete velikost síly F podle obrázku



$M = -3690\,Ncm$

$b = 330\,mm$

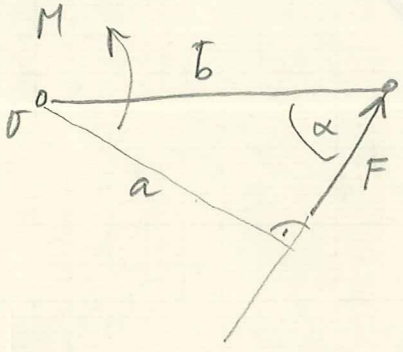
$\alpha = 41°$

$$F_y = F\sin\alpha$$

$$M = F_y.b \quad \therefore \quad F_y = \frac{M}{b} = \frac{-36900}{330} = 111,8$$

$$F = \frac{F_y}{\sin\alpha} = \frac{111,8}{\sin 41°} = \underline{170,4\,N}$$

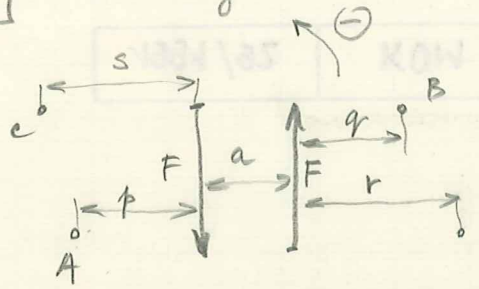
1.26 Určete vzdálenost b síly $F = 58\,N$ od pevného bodu O síla F je sklonena pod úhlem $\alpha = 53°$ a $M = 41202\,Nmm$



$$a = \frac{M}{F} = \frac{41202}{58} = 710,37\,mm$$

$$b = \frac{a}{\sin\alpha} = \frac{710,37}{\sin 53°} = \underline{889,47\,mm}$$

1.27 Dvěma silami $F = 920\,N$ jsou vzdáleny $a = 410\,mm$; p, q, r, s. ⊖

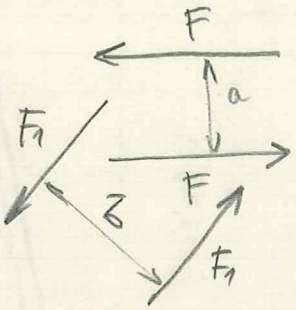


Máme dokázat, že dvojice sil musíme v prostoru libovolně posouvat a otáčet aniž se účinek změní

$A \dots F.p - F(a+p) = F(p-a-p) = -F.a$

$p = 160\,mm$; $q = 380$; $r = 580$; $s = 1670$ | atd.

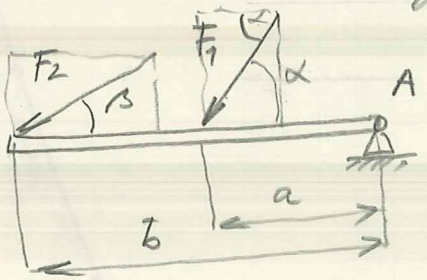
1.28 Dvojice sil $F = 46\,N$ a rameno $a = 390\,mm$ jaké rameno $b = ?$ musí mít dvojice sil $F_1 = 63\,N$, při stejném účinku.



$$M = F.a = F_1 b$$

$$b = \frac{F}{F_1} a = \frac{46}{63} 390 = \underline{284,7\,mm}$$

1.29 Na páce podle obr. je $F_1 = 780\,N$ působící pod $\alpha = 48°$ ve vzdálenosti $a = 150\,mm$. Jak velká je $F_2 = ?$ která nahradí otáčivý účinek F_1 při $b = 220\,mm$ a $\beta = 26°$



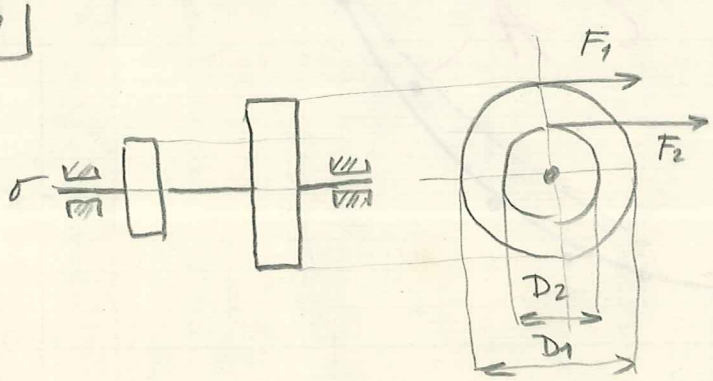
$$F_{1y} = F_1 \sin\alpha = 780 \sin 48° = \underline{579,6\,N}$$

$$M = F_{1y}.a = F_{2y}.b$$

$$F_{2y} = F_{1y} \frac{a}{b} = 579,65 \frac{150}{220} = \underline{395,21}$$

$$F_2 = \frac{F_{2y}}{\sin\beta} = \frac{395,21}{\sin 26°} = \underline{901,54\,N}$$

1.30

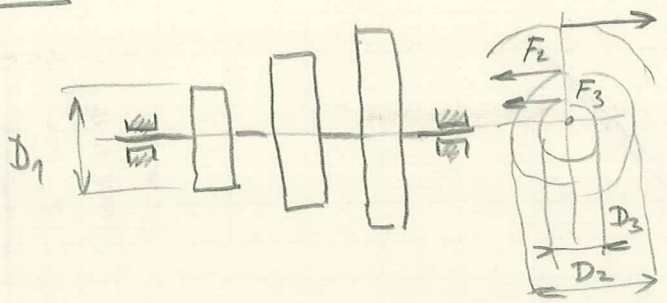


Na řemenici $D_1 = 960\,mm$ působí síla $F_1 = 350\,N$. Jaká hnací síla $F_2 = ?$ na $D_2 = 730\,mm$ bude mít stejný otáčivý účinek?

$$M = F_1 \cdot \frac{D_1}{2} = F_2 \cdot \frac{D_2}{2}$$

$$F_2 = F_1 \frac{\frac{D_1}{2}}{\frac{D_2}{2}} = 350 \frac{960}{730} = \underline{460,27\,N}$$

1.31 Na hřídeli jsou řemenice $D_1 = 670\,mm$, $D_2 = 520$, $D_3 = 390$; $F_1 = 165\,N$, $F_2 = ?$, $F_3 = ?$



F_2, F_3 – hodnoty sil, které zaměří otáčení jestliže $F_2 : F_3 = 3:5$

$$F_1 \cdot \frac{D_1}{2} = F_2 \frac{D_2}{2} + F_3 \frac{D_3}{2} \quad /\cdot 2$$

$$\frac{F_2}{F_3} = \frac{3}{5} \dots F_2 = \frac{3}{5} F_3$$

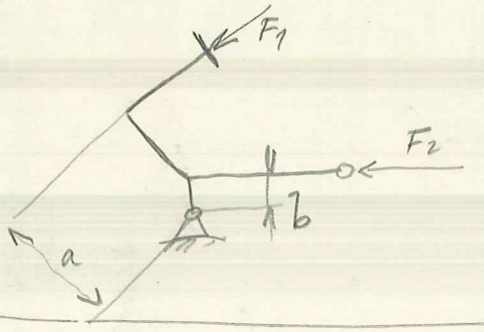
$F_1 D_1 = F_2 D_2 + F_3 D_3$ – jedna rovnice 2 neznámé, druhá rovnice $F_2 = \frac{3}{5} F_3$

$$F_1 D_1 = \frac{3}{5} F_3 D_2 + F_3 D_3$$

$$F_3 = \frac{F_1 D_1}{\left(\frac{3}{5} D_2 + D_3\right)} = \frac{165 \cdot 670}{\frac{3}{5} 520 + 390} = 157{,}47\,N$$

$$F_2 = \frac{3}{5} F_3 = \frac{3}{5} \cdot 157{,}47 = 94{,}48\,N$$

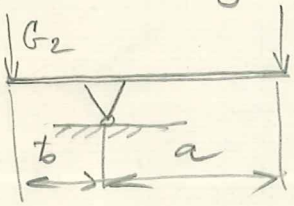
1.32 Na pedál brzdy působí síla $F_1 = 160\,N$. Jakou silou F_2 působí pedál na rovnoměrné $F_2 = ?$ $a = 320\,mm$, $b = 80\,mm$



$$F_1 a = F_2 b$$

$$F_2 = F_1 \frac{a}{b} = 160 \frac{320}{80} = 640\,N$$

1.33 Jakou silou ~~vyvodí~~ rovnám zvedákem dítě, které váží 80 kg. $G_2 = ?$ $a = 320\,mm$, $b = 80\,mm$

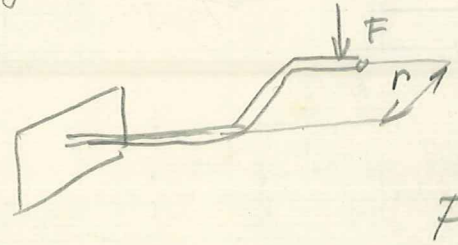


$$G_1 = mg = 80 \cdot 9{,}81 = 784{,}8\,N$$

$$G_1 \cdot a = G_2 b$$

$$G_2 = G_1 \frac{a}{b} = 784{,}8 \frac{320}{80} = 3139{,}2\,N$$

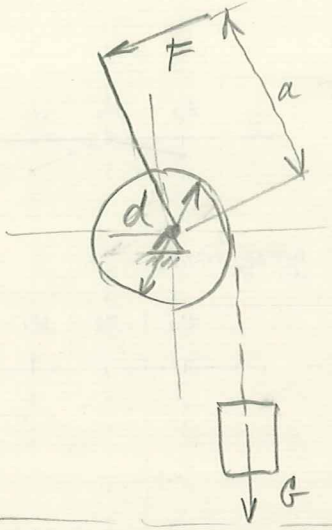
1.34 Jakou silou musíme působit klíčem motoru když je potřebný moment $M = 98\,000\,Nmm$, $r = 28\,cm$



$$F = \frac{M}{r} = \frac{98000}{280} = 350\,N$$

1.35 Jaká síla je potřebná na rumpále $G = 1200\,N$

$d = 200\,mm$; $a = 450\,mm$



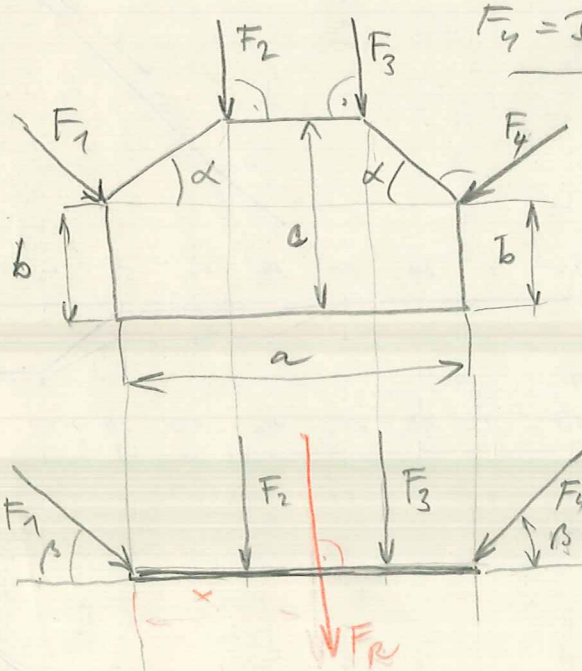
$$F.a = G.\frac{d}{2}$$

$$F = \frac{G}{a}\frac{d}{2} = \frac{1200}{450}\frac{200}{2} = \underline{\underline{266,6\,N}}$$

1.36 Najděte výslednici sil F, $a = 750\,mm$; $b = 250\,mm$; $c = 450$

$\alpha = 45°$; $F_1 = 380\,N$; $F_2 = 210\,N$; $F_3 = 270\,N$;

$F_4 = 350\,N$



$\beta = 45°$

$F_{1x} = F_1 \cos\beta = 380 \cos 45° = \underline{268,7\,N}$

$F_{1y} = F_1 \sin\beta = \underline{268,7\,N}$

$F_{4x} = F_4 \cos\beta = 350 \cdot \cos 45° = \underline{247,45\,N}$

$F_{4y} = F_4 \sin\beta = \underline{247,45\,N}$

x ... $F_{Rx} = F_{1x} - F_{4x} = 268,7 - 247,4 = 21,3$

y ... $F_{Ry} = F_{1y} + F_2 + F_3 + F_{4y} =$

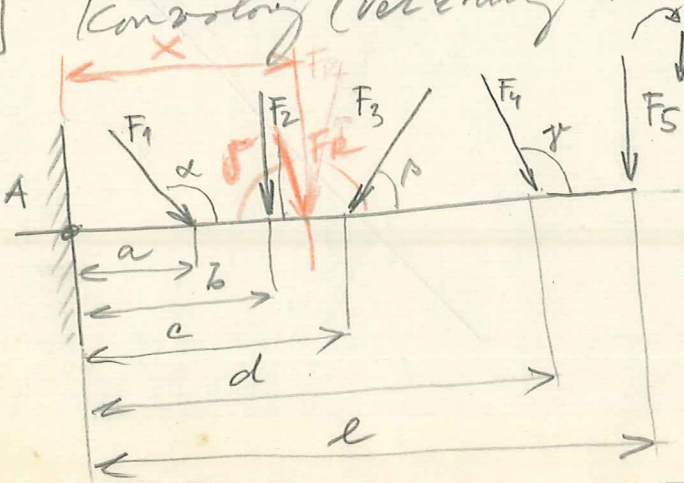
$= 268,7 + 210 + 270 + 247,4 =$

$= \underline{\underline{996,1\,N}}$

$$F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2} =$$

$$= \sqrt{21,3^2 + 996,1^2} = \underline{\underline{996,32\,N}}$$

1.37 Konzola (vetknutý nosník) je zatížen podle obr.



$F_1 = 180\,N$

$F_2 = 210\,N$

$F_3 = 240\,N$

$F_4 = 260\,N$

$F_5 = 190\,N$

$a = 190\,mm$

$b = 310\,mm$

$c = 480\,mm$

$d = 630\,mm$

$e = 850\,mm$

$\alpha = 135°$

$\beta = 60°$

$\gamma = 100°$

$\underline{F_R\,;\,x\,;\,\delta = ?}$

1,37

$x = \sqrt{16}$

$\delta = 93°$

F_1 … $F_{1x} = F_1 \cos\alpha' = 180 \cdot \cos 45° = \underline{127{,}2\,N}$

$F_{1y} = \underline{127{,}2\,N}$

$\alpha' = 180 - \alpha = 180 - 135 = 45°$

F_2 … $F_{2x} = 0$

$F_{2y} = F_2 = 210\,N$

F_3 … $F_{3x} = F_3 \cos\beta = 240 \cdot \cos 60° = \underline{120\,N}$

$F_{3y} = F_3 \sin\beta = 240 \cdot \sin 60° = \underline{207{,}8\,N}$

$\gamma' = 180 - \gamma = 180 - 100 = 80°$

F_4 … $F_{4x} = F_4 \cos\gamma' = 260 \cos 80° = \underline{45{,}1\,N}$

$F_{4y} = F_4 \sin\gamma' = 260 \cdot \sin 80° = \underline{256\,N}$

F_5 … $F_{5x} = 0$

$F_{5y} = F_5 = \underline{190\,N}$

---

x: $F_{Rx} = \Sigma F_{xi}$ | $F_{Rx} = F_{1x} + F_{2x} - F_{3x} + F_{4x} + F_{5x} = 127{,}2 + 0 - 120 + 45{,}1 + 0 = \underline{61{,}3}$

y: $F_{Ry} = \Sigma F_{yi}$ | $F_{Ry} = F_{1y} + F_{2y} + F_{3y} + F_{4y} + F_{5y} = 127{,}2 + 210 + 207{,}8 + 256 + 190 = \underline{991\,N}$

$$F_R = \sqrt{F_{1x}^2 + F_{2y}^2} = \sqrt{61{,}3^2 + 991^2} = \underline{992{,}9\,N}$$

M: $F_R \cdot x = \Sigma M_i$ | $F_R \cdot x = F_{1y} \cdot a + F_{2y} \cdot b + F_{3y} \cdot c + F_{4y} \cdot d + F_{5y} \cdot e$

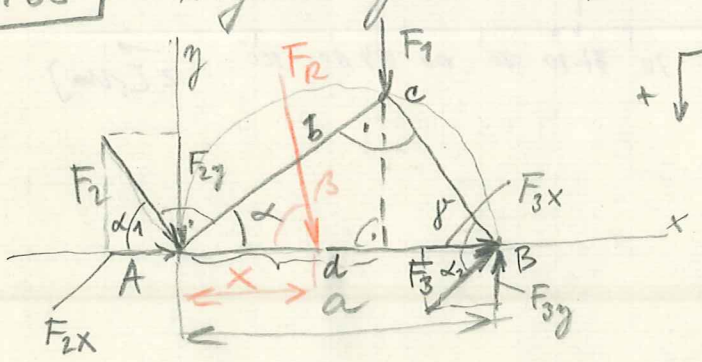
$$x = \frac{127{,}2 \cdot 190 + 210 \cdot 310 + 207{,}8 \cdot 480 + 256 \cdot 630 + 190 \cdot 850}{992{,}9}$$

$$= \frac{24168 + 65100 + 99744 + 161280 + 161500}{992{,}9} = \underline{515{,}45\,mm}$$

$$tg\,\delta' = \frac{F_{Ry}}{F_{Rx}} = \frac{991}{61{,}3} = 16{,}166 \dots \delta' = 86{,}46$$

$\delta = 180 - \delta' = 180 - 86{,}46 = \underline{93{,}54}$

1,38 Najděte výslednici F_R této síl a vzdálenost x od bodu A a úhel β mezi F_R a přímkou AB



$F_1 = 860\,N$

$F_2 = 1200\,N$

$F_3 = 620\,N$

$a = 800\,mm$

$\alpha = 30°$

1,38

$F_{1x} = 0$

$F_{1y} = F_1 = 860\,N$

$F_{2x} = F_2 \cos\alpha_1 = 1200 \cos 60° = 600\,N$

$F_{2y} = F_2 \sin\alpha_1 = 1200 \sin 60° = 1039{,}2\,N$

$F_{3x} = F_3 \cos\alpha_2 = 620 \cos 60° = 310\,N$

$F_{3y} = F_3 \sin\alpha_2 = 620 \sin 60° = 536{,}9\,N$

$\alpha_1 = 180 - 90 - \alpha = 180 - 90 - 30 = 60$

$\gamma = 180 - 90 - \alpha = 60°$

$\alpha_2 = 90 - \gamma = 30°$

---

x: $F_{Rx} = \sum F_{ix}$

$F_{Rx} = 0 + F_{2x} + F_{3x} = 600 + 310 = \underline{910\,N}$

y: $F_{Ry} = \sum F_{iy}$

$F_{Ry} = F_{1y} + F_{2y} - F_{3y} = 860 + 1039{,}2 - 536{,}9 = \underline{1362{,}3\,N}$

$F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2} = \sqrt{910^2 + 1362{,}3^2} = \underline{1638{,}2\,N}$

$\operatorname{tg}\beta = \frac{F_{Ry}}{F_{Rx}} = \frac{1362{,}3}{910} = 1{,}497 \quad \Rightarrow \underline{\beta = 56{,}25°}$

M_A: $F_{Ry} \cdot X = \sum M_{Ai}$

$X = \frac{+F_1 \cdot d + F_{3y} \cdot a}{F_{Ry}} = \frac{860 \cdot 600 - 536{,}9 \cdot 800}{1362{,}3}$

$= \underline{63{,}48\,mm}$

$\frac{b}{a} = \cos\alpha \rightarrow b = a\cos\alpha$

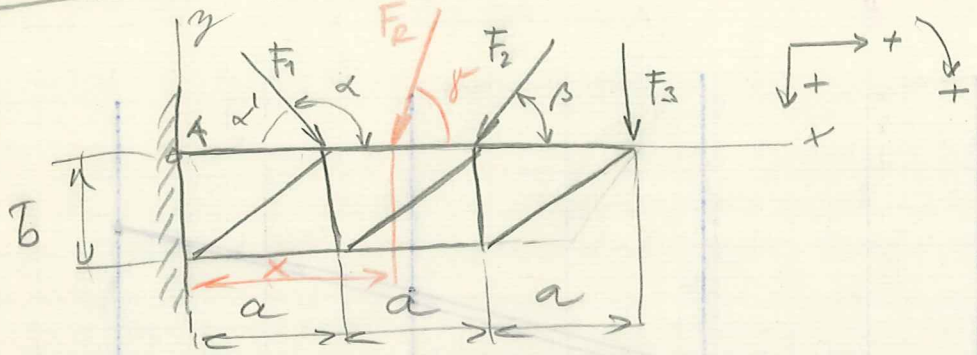
$\frac{d}{b} \cos\alpha \rightarrow d = b\cos\alpha$

$d = a\cos\alpha \cdot \cos\alpha =$

$= a\cos^2\alpha = 800 \cos^2 30° = 600$

Průsvitek – sčítání F_1 F_2

1,39 | Na traverzu konstrukce podle obr.



$F_1 = 900\,N$
$F_2 = 1400\,N$
$F_3 = 1100\,N$
$F_4 = ?$ v rovnováze
$\alpha = 120°$
$\beta = 45°$
$a = 380\,mm$
$b = 560\,mm$
$x = ?$
$\gamma = ?$

$F_{1x} = F_1 \cos\alpha' = 900 \cos 60° = 450\,N$
$F_{1y} = F_1 \sin\alpha' = 900 \sin 60° = 779{,}4\,N$
$F_{2x} = F_2 \cos\beta = 1400 \cdot \cos 45° = 989{,}9\,N$
$F_{2y} = F_2 \sin\beta = 1400 \sin 45° = 989{,}9\,N$
$F_{3x} = 0$
$F_{3y} = F_3 = 1100\,N$

x = 800
γ 80°

X: $F_{Rx} = \sum F_{ix}$
$F_{Rx} = F_{1x} - F_{2x} = 450 - 989{,}9 = -539{,}9$ jde doleva

y: $F_{Ry} = \sum F_{iy}$
$F_{Ry} = F_{1y} + F_{2y} + F_3 = 779{,}4 + 989{,}9 + 1100 = 2869{,}3\,N$

$F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2} = \sqrt{(-539{,}9)^2 + 2869{,}3^2} = 2919{,}6\,N$

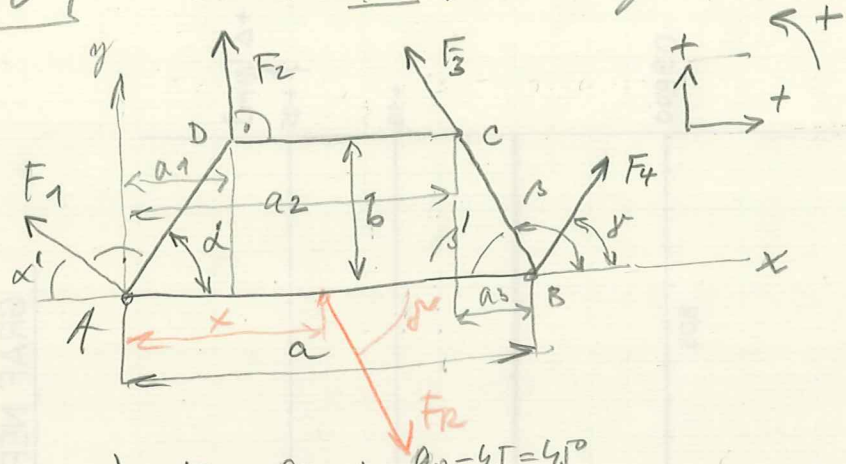
$\operatorname{tg}\gamma = \frac{F_{Ry}}{F_{Rx}} = \frac{2869{,}3}{539{,}9} = 5{,}314$ $\quad \gamma = 79{,}34°$

M_A: $F_{Ry} \cdot x = F_{1y} \cdot a + F_{2y} \cdot 2a + F_3 \cdot 3a$

$$x = \frac{a(F_{1y} + 2F_{2y} + 3F_3)}{F_{Ry}} = \frac{380(779{,}4 + 2 \cdot 989{,}9 + 3 \cdot 1100)}{2869{,}3}$$

$= 802{,}45\,mm$

1:40 | Určete F_5 složky výsledné síly na kliku



$F_1 = 5100\,N$
$F_2 = 3400\,N$
$F_3 = 4600\,N$
$F_4 = 3900\,N$
$\alpha = 45°$
$\beta = 120°$
$\gamma = 60°$
$a = 610\,mm$
$b = 260\,mm$

$\alpha' = 180 - 90 - \alpha = 90 - 45 = 45°$

$\beta' = 180 - 120 = 60°$

$F_{1x} = F_1 \cos\alpha' = 5100 \cos 45° = 3605{,}7\,N$

$F_{1y} = F_1 \sin\alpha' = 3605{,}7$

$F_{2x} = 0$

$F_{2y} = F_2 = 3400\,N$

$F_{3x} = F_3 \cos\beta' = 4600 \cos 60° = 2300\,N$

$F_{3y} = F_3 \sin\beta' = 4600 \sin 60° = 3983{,}7\,N$

$F_{4x} = F_4 \cos\gamma = 3900 \cos 60° = 1950\,N$

$F_{4y} = F_4 \sin\gamma = 3900 \sin 60° = 3377{,}4$

x: $\Sigma F_{xi} = 0$

$$F_{5x} - F_{1x} + F_{2x} - F_{3x} + F_{4x} = 0$$

$$F_{5x} = F_{1x} - F_{2x} + F_{3x} - F_{4x} = 3605{,}7 - 0 + 2300 - 1950 = 3955{,}7\,N$$

y: $\Sigma F_{yi} = 0$

$$-F_{5y} + F_{1y} + F_{2y} + F_{3y} + F_{4y} = 0$$

$$F_{5y} = F_{1y} + F_{2y} + F_{3y} + F_{4y} = 3605{,}7 + 3400 + 3983{,}7 + 3377{,}4 = 14366{,}8$$

$$F_5 = \sqrt{F_{5x}^2 + F_{5y}^2} = \sqrt{3955{,}7^2 + 14366{,}8^2} = 14901{,}4\,N$$

$$\operatorname{tg}\delta = \frac{F_{5y}}{F_{5x}} = \frac{14366{,}8}{3955{,}7} = 3{,}631 \qquad \delta = 74{,}6°$$

1.40

$M_A - \sum M_i = 0 \quad -F_{5y} \cdot x + F_{2y} \cdot a_1 + F_{3y} \cdot a_2 + F_{4y} \cdot a = 0$

$\frac{b}{a_1} \operatorname{tg}\alpha$

$a_1 = \frac{b}{\operatorname{tg}\alpha} = \frac{260}{\operatorname{tg}45°} = 260\ m$

$x = \frac{F_{2y} \cdot a_1 + F_{3y} \cdot a_2 + F_{4y} \cdot a}{F_{5y}}$

$a_2 = a - a_3 = 610 - 150,1 = 459,$

$a_3 = \frac{b}{\operatorname{tg}\beta} = \frac{260}{\operatorname{tg}60°} = 150,1$

$= \frac{3400 \cdot 260 + 3983,7 \cdot 459,9 + 3377,4 \cdot 610}{14\,366,8}$

$= \frac{884000 + 1832\,103,6 + 2060214}{14\,366,8} = 332,45\ mm$

1.41 | Určete graficky výslednici F_R pětice

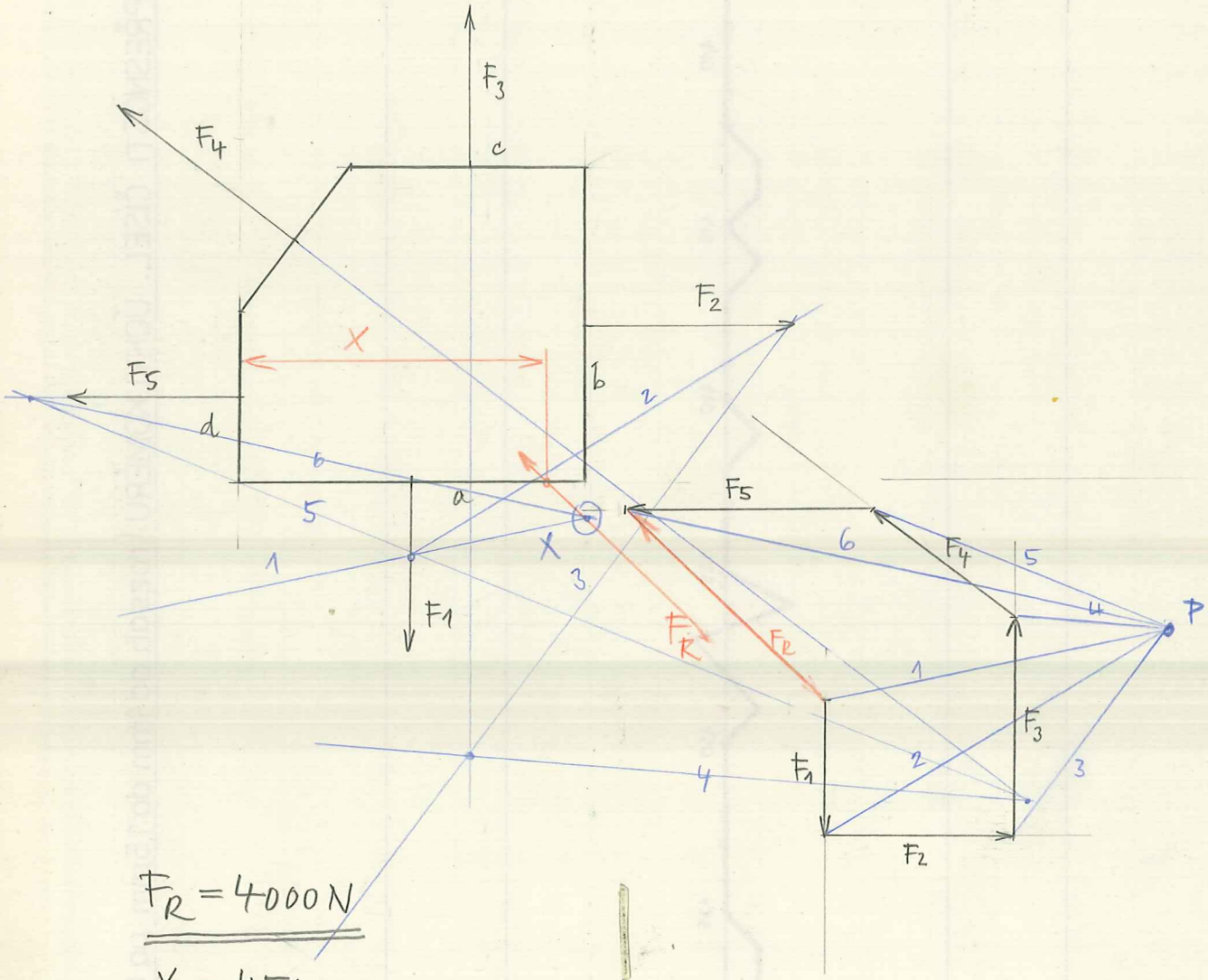
$m_L \Rightarrow 1\,mm \mathrel{\hat{=}} 10\,mm$

$m_F \Rightarrow 1\,mm \mathrel{\hat{=}} 100\,N$

$F_1 = 2000\,N$
$F_2 = 2800\,N$
$F_3 = 3200\,N$
$F_4 = 2600\,N$
$F_5 = 3700\,N$

$\alpha = ?$
$F_R = ?$
$x = ?$ AB

$a = 510\,mm$
$b = 460\,mm$
$c = 340\,mm$
$d = 250\,mm$



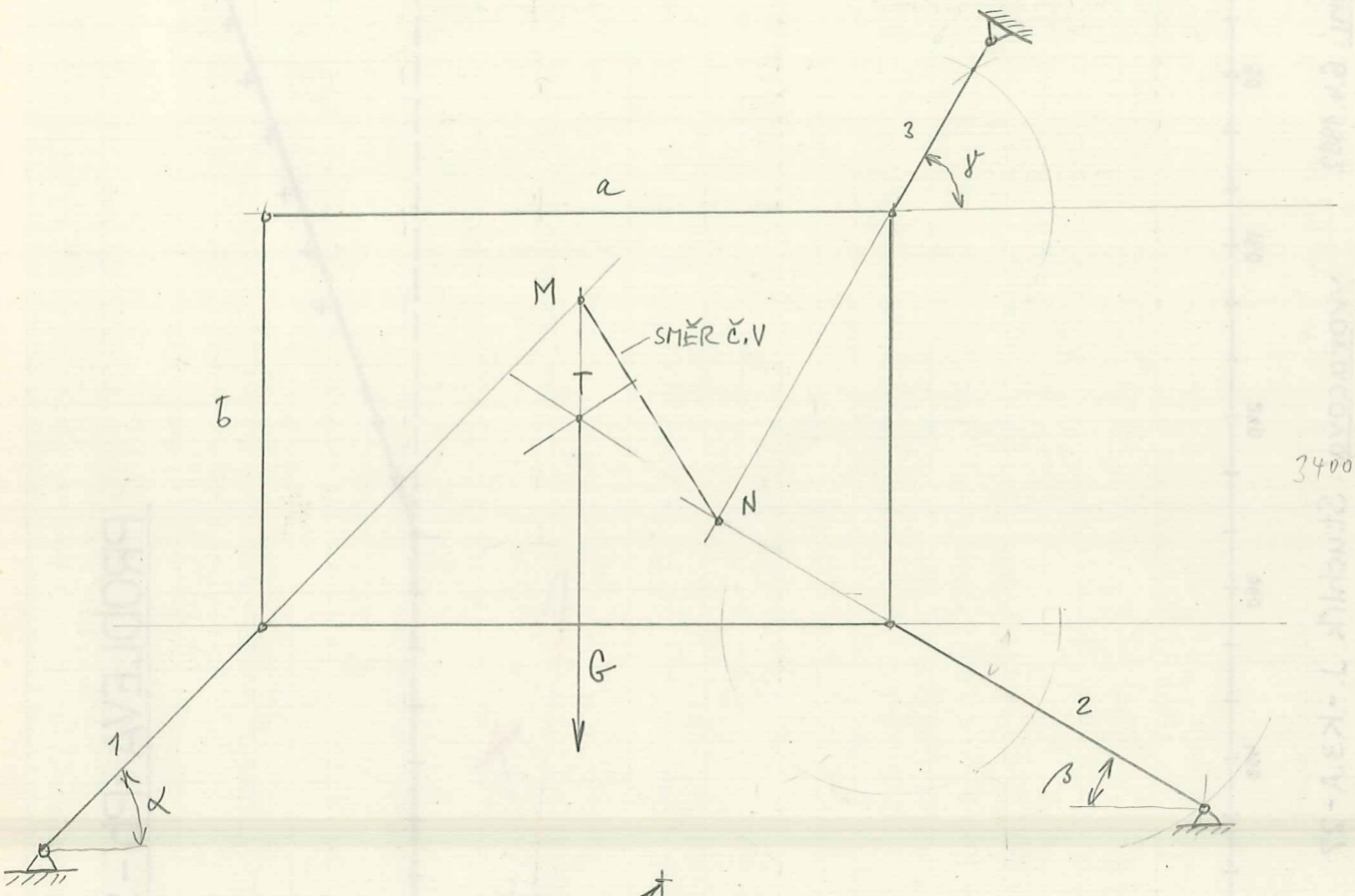
$F_R = 4000\,N$

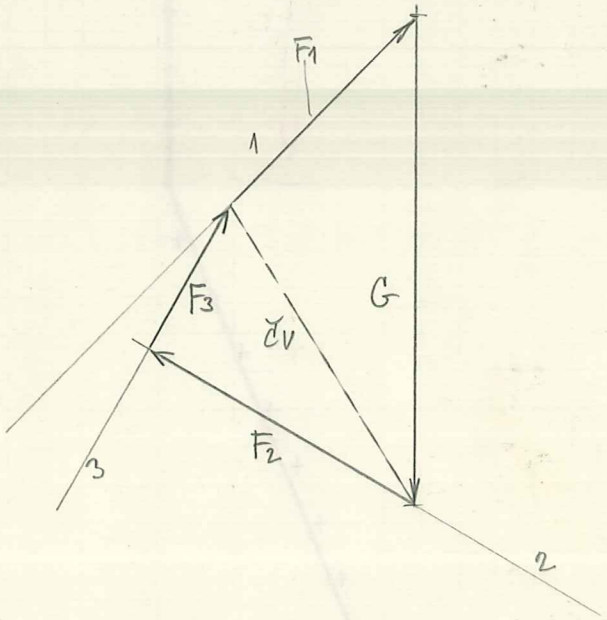
$X = 450\,mm$

$\alpha = 44°$

1.42 Ocel. tyč o $G = 3200\,N$ je v rovnováze podle obr.
Graficky určete F_1, F_2, F_3;

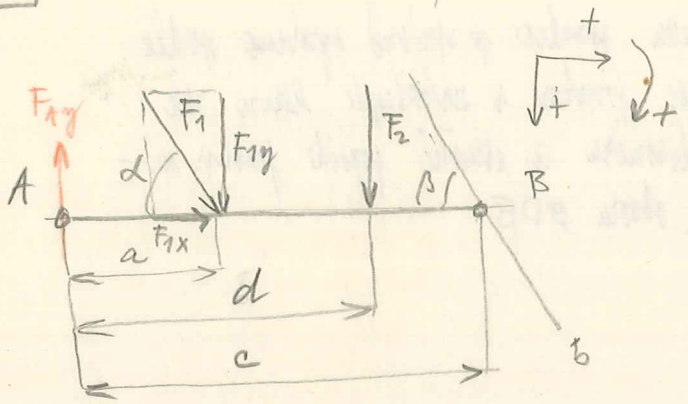
$a = 420\,mm$
$b = 270\,mm$
$\alpha = 45°$
$\beta = 30°$; $\gamma = 60°$





$F_1 = 1700\,N$
$F_2 = 2000\,N$
$F_3 = 1000\,N$

1.45



$F_1 = 920\,N$ $\quad F_A = ?$

$F_2 = 660\,N$ $\quad F_B = ?$: ROVNOVÁHA

$a = 280\,mm$

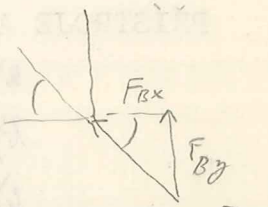
$d = 410\,mm$

$c = 630\,mm$

$\alpha = 60°$

$\beta = 45°$

$$F_{1x} = F_1 \cos\alpha = 920 \cos 60° = 460\,N$$
$$F_{1y} = F_1 \sin\alpha = 920 \sin 60° = 796{,}7\,N$$



$$tg\,\beta = \frac{F_{By}}{F_{Bx}}$$

$X:\ -\Sigma F_{ix} = 0 \ \ldots\ F_{Ax} + F_{1x} - F_{Bx} = 0$

$y:\ -\Sigma F_{iy} = 0 \ \ldots\ F_{Ay} + F_{1y} + F_{2y} - F_{By} = 0$

$M_A:\ \Sigma M_i = 0 \ \ldots\ F_{1y} \cdot a + F_2 \cdot d - F_{By} \cdot c = 0$

$$F_{By} = \frac{F_{1y} \cdot a + F_2 d}{c} = \frac{796{,}7 \cdot 280 + 660 \cdot 410}{630}$$
$$= \underline{783{,}6\,N}$$

$$F_{Ay} = F_{By} - F_{1y} - F_{2y} = 783{,}6 - 796{,}7 - 660 = \underline{-673{,}1\,N}$$ jde nahoru

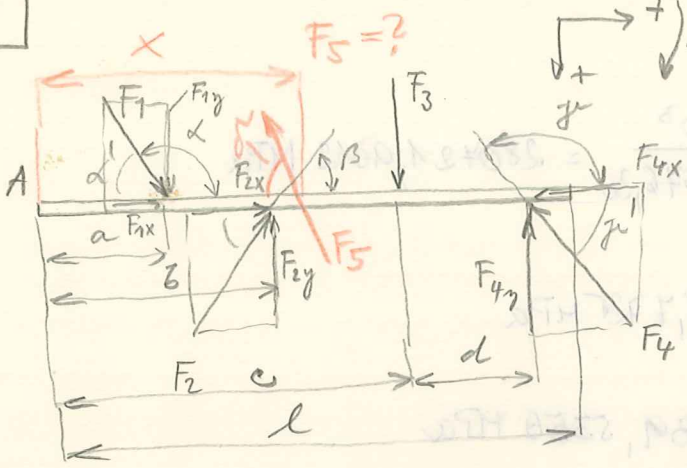
$$F_{Bx} = \frac{F_{By}}{tg\,\beta} = \frac{783{,}6}{tg\,45°} = \underline{783{,}6}$$

$$F_{Ax} = F_{Bx} - F_{1x} = 783{,}6 - 460 = \underline{323{,}6\,N}$$

$$F_A = \sqrt{F_{Ax}^2 + F_{Bx}^2} = \sqrt{323{,}6^2 + (-673{,}1)^2} = \underline{746{,}8\,N}$$

$$F_B = \sqrt{F_{Bx}^2 + F_{By}^2} = \sqrt{783{,}6^2 + 783{,}6^2} = \underline{1108\,N}$$

1.46



$F_1 = 960\,N$ $\quad F_5 = ?$
$F_2 = 750\,N$ $\quad x = ?$
$F_3 = 820\,N$ $\quad \alpha_5 = ?$
$F_4 = 1100\,N$

$a = 250\,mm$ $\quad \alpha = 120°$
$b = 420\,mm$ $\quad \beta = 45°$
$c = 690\,mm$ $\quad \gamma = 150°$
$l = 1200\,mm$
$d = 240\,mm$

ROVNOVÁHA

$\alpha' = 180 - \alpha = 60°$
$\gamma' = 180 - \gamma = 30°$

$F_{1x} = F_1 \cos\alpha' = 960 \cos 60° = \underline{480\,N}$
$F_{1y} = F_1 \sin\alpha' = 960 \sin 60° = \underline{831{,}4\,N}$
$F_{2x} = F_2 \cos\beta = 750 \cos 45 = \underline{530{,}3}$
$F_{2y} = F_{2x} = \underline{530{,}3\,N}$
$F_{3x} = 0$
$F_{3y} = F_3$
$F_{4x} = F_4 \cos\gamma' = 1100 \cdot \cos 30° = \underline{952{,}6\,N}$
$F_{4y} = F_4 \sin\gamma' = 1100 \cdot \sin 30° = \underline{550\,N}$

$x \dots \Sigma F_{xi} = 0 \dots F_{1x} + F_{2x} - F_{4x} + F_{5x} = 0$
$y \dots \Sigma F_{yi} = 0 \dots F_{1y} - F_{2y} + F_3 - F_{4y} + F_{5y} = 0$
$M_A \dots \Sigma M_i = 0 \dots F_{1y} \cdot a - F_{2y} b + F_3 c - F_{4y} \cdot (c+d) + F_{5y} \cdot x = 0$

$$x = \frac{-F_{1y} a + F_{2y} b - F_3 c + F_{4y}(c+d)}{F_{5y}} =$$

$$= \frac{-831{,}4 \cdot 250 + 530{,}3 \cdot 420 - 820 \cdot 690 + 550 \cdot (690 + 240)}{-571{,}1}$$

$F_{5y} = -F_{1y} + F_{2y} - F_3 + F_{4y} = -831{,}4 + 530{,}3 - 820 + 550 = \underline{-571{,}1}$

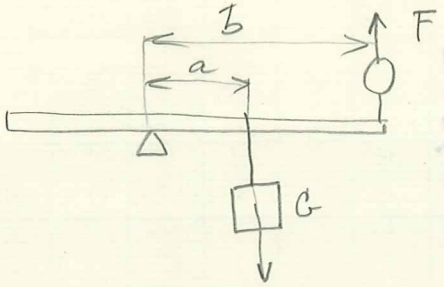
síla jde nahoru předpoklad nesprávný.

$$x = \frac{-207850 + 222726 - 565800 + 571500}{-571{,}1} = \frac{-39424}{-571{,}1} = \underline{+69\,mm}$$

$F_{5x} = F_{4x} - F_{1x} - F_{2x} = 952{,}6 - 480 - 530{,}3 = \underline{-57{,}7\,N}$

$F_5 = \sqrt{F_{5x}^2 + F_{5y}^2} = \sqrt{57{,}7^2 + 571{,}1^2} = \underline{574\,N}$ $\quad tg\,\delta = \frac{F_{5y}}{F_{5x}} = 9{,}897$ $\quad \underline{\delta = 84{,}23°}$

1.47



$G = 17\,N$
$b = 186\,mm$
$a = 125\,mm$

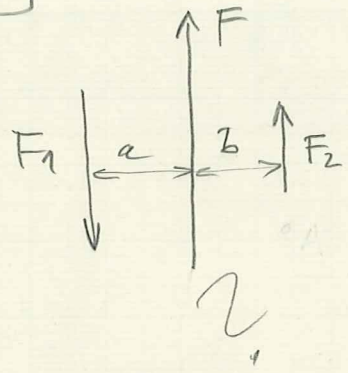
Rovnováha

$\Sigma M_i = 0$

$F.b - G.a = 0$

$F = G\frac{a}{b} = 17\frac{125}{186} = \underline{\underline{11{,}4\,N}}$

1.48

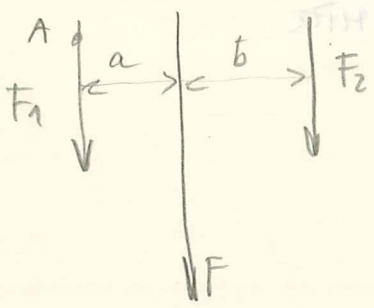


$F = 47\,N$ je výsledná F_1 a F_2
$F_1 = 152\,N$; $(a+b) = 480\,mm$
$F_2 = ?$
$b = ?$

$-F_1 + F_2 = F$

$F_2 = F + F_1 = 47 + 152 = \underline{\underline{199\,N}}$

1.49



$F = 3510\ N$ je výslednice F_1 a F_2

$F_1 = 1860\ N$; $a = 580\ mm$; $F_2 = ?$; $b = ?$

$y: \quad -\Sigma F_i = F_R \quad \dots \quad F_1 + F_2 = F_R$

$F_2 = F_R - F_1 = 3510 - 1860$

$= \underline{1650\ N}$

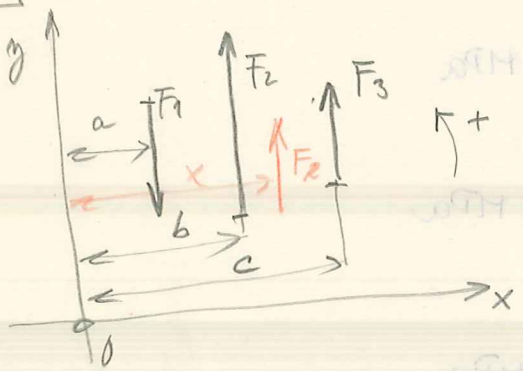
$M_A: \quad -\Sigma M_i = F_R \cdot a$

$F_2 (a+b) = F_R \cdot a$

$F_2 a + F_2 b = F_R \cdot a$

$b = \frac{a(F_R - F_2)}{F_2} = \frac{580 \cdot 1860}{1650} = \underline{653,9\ mm}$

1.50 ④



$F_1 = 740\ N$

$F_2 = 480\ N$

$F_3 = 560\ N$

$a = 250\ mm$

$b = 430\ mm$

$c = 590\ mm$

$F_R = ?$

$x = ?$

$y: \quad -F_1 + F_2 + F_3 = F_R$

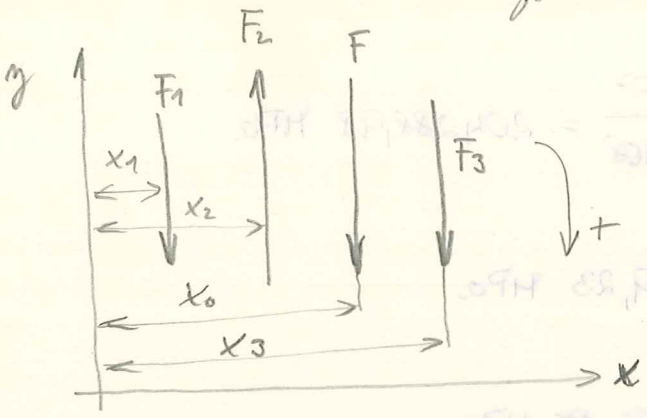
$F_R = -740 + 480 + 560 = \underline{300\ N} \uparrow$

$M_0: \quad -\Sigma M_i = F_R \cdot x$

$F_R \cdot x = -F_1 a + F_2 b + F_3 c$

$x = \frac{-740 \cdot 250 + 480 \cdot 430 + 560 \cdot 590}{300} = \underline{1172,6\ N}$

1.51



Vyjedmice $-F = 180\,N$
$F_1 = 320\,N$
$F_2 = 430\,N$
$F_3 = ?$
$x_3 = ?$

$x_1 = 230\,mm$
$x_2 = 380\,mm$
$x_0 = 470\,mm$

$y: \Sigma F_i = F$

$$-F = -F_1 + F_2 - F_3$$

$$F_3 = F - F_1 + F_2 = 180 - 320 + 430 = \underline{290\,N}$$

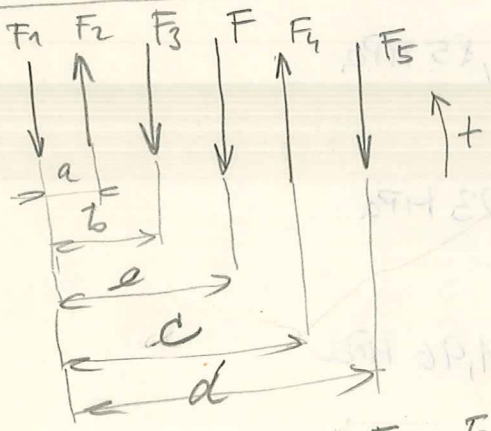
$M: \Sigma M_0 = F \cdot x_0$

$$F_1 x_1 - F_2 x_2 + F_3 x_3 = F \cdot x_0$$

$$x_3 = \frac{F x_0 + F_2 x_2 - F_1 x_1}{F_3} = \frac{180 \cdot 470 + 430 \cdot 380 - 320 \cdot 230}{290}$$

$$= \frac{248000 - 73600}{290} = \underline{601{,}4\,mm}$$

1.52



$F_1 = 890\,N$
$F_2 = 930\,N$
$F_3 = 720\,N$
$F_4 = 640\,N$
$F_5 = 820\,N$

$F = ?$ / $e = ?$
$a = 180\,mm$
$b = 280\,mm$
$c = 390\,mm$
$d = 520\,mm$

$X: -F_1 + F_2 - F_3 + F_4 - F_5 = -F$

$$F = F_1 - F_2 + F_3 - F_4 + F_5 = 890 - 930 + 720 - 640 + 820 = \underline{860}$$

$M: \Sigma M_i = -F \cdot e$

$$-F \cdot e = F_2 a - F_3 b + F_4 c - F_5 d$$

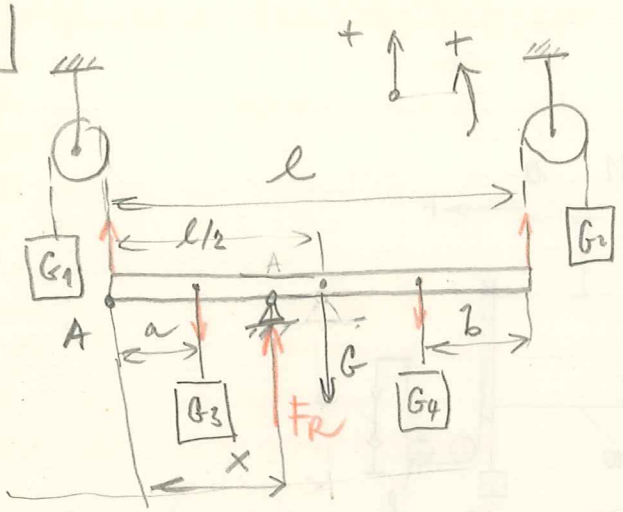
$$e = \frac{F_2 a - F_3 b + F_4 c - F_5 d}{-F} =$$

$$= \frac{930 \cdot 130 - 720 \cdot 280 + 640 \cdot 390 - 820 \cdot 520}{-860} =$$

$$= \frac{120900 - 201600 + 249600 - 426400}{-860} = \frac{-257500}{-860} =$$

$$= \underline{299{,}4\,mm}$$

1.53



$G = 35\,N$
$l = 520\,mm$
$G_3 = 18\,N$
$G_4 = 27\,N$
$G_1 = 30\,N$
$G_2 = 25\,N$

$x = ?$ RAVNOVESJE

$a = 140\,mm$
$b = 170\,mm$

$y: \quad \Sigma F_{iy} = 0$

$$G_1 - G_3 - G_4 + G_2 - G + F_R = 0$$

$$F_R = -G_1 + G_3 + G_4 - G_2 + G = -30 + 18 + 27 - 25 + 35 = \underline{25\,N}$$

$M_A: \quad \Sigma M_i = 0$

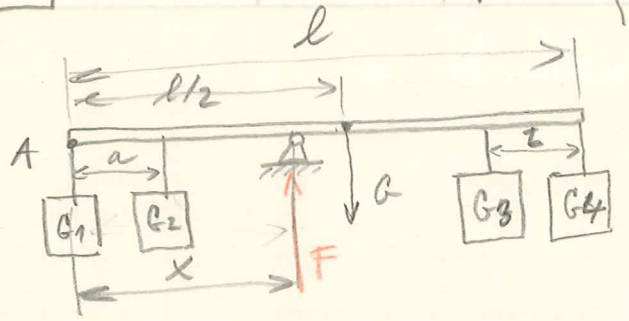
$$-G_3 a + F_R x - G\frac{l}{2} - G_4(l-b) + G_2 l = 0$$

$$x = \frac{G_3 a + G\frac{l}{2} + G_4(l-b) - G_2 l}{F_R} =$$

$$= \frac{18 \cdot 140 + 35 \cdot 260 + 27 \cdot 350 - 25 \cdot 520}{25} =$$

$$= \frac{2520 + 9100 + 9450 - 13000}{25} = \underline{322{,}8\,mm}$$

1.54



$G = 140\,N$ $\quad l = 230\,cm$

$G_1 = 30\,N$ $\quad a = 65\,cm$

$G_2 = 120\,N$ $\quad b = 84\,cm$

$G_3 = 90\,N$

$G_4 = 130\,N$

ROVNOVÁHA

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y: $\Sigma F_{iy} = 0$

$$G_1 + G_2 - F + G + G_4 + G_3 = 0$$

$$F = G_1 + G_2 + G + G_3 + G_4 = 30 + 120 + 140 + 90 + 130 = \underline{510\,N}$$

M_A: $\Sigma M_{iA} = 0$

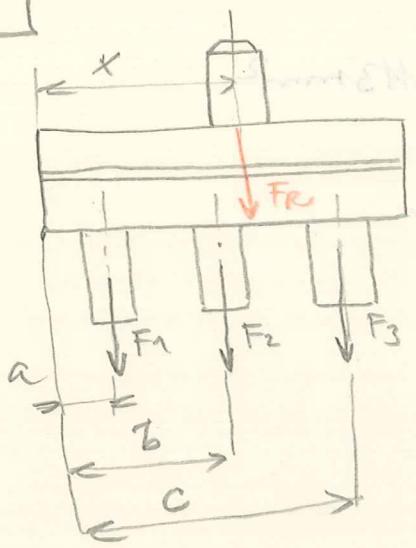
$$-G_2 \cdot a + F \cdot x - G\frac{l}{2} - G_3(l-b) - G_4 l = 0$$

$$x = \frac{G_2 a + G\frac{l}{2} + G_3(l-b) + G_4 l}{F} =$$

$$= \frac{120 \cdot 65 + 140 \cdot 115 + 90 \cdot 146 + 130 \cdot 230}{510} =$$

$$= \frac{7800 + 16100 + 13140 + 29900}{510} = \underline{131{,}2\,cm}$$

1.55



$F_1 = 5600\ N$
$F_2 = 8900\ N$
$F_3 = 6300 N$
$a = 150\ mm$
$b = 330\ mm$
$c = 440\ mm$

$F = ?$
$x = ?$

y: $\quad -\!\!- \quad \Sigma F_i = F_R$

$F_1 + F_2 + F_3 = F_R$

$F_R = 5600 + 8900 + 6300 = 20\,800\ N$

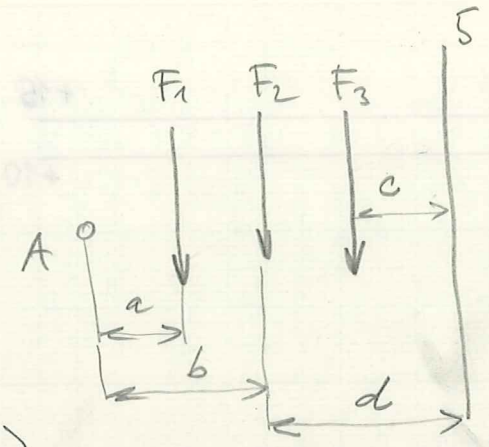
M: $\quad F_R \cdot x = \Sigma M_i$

$F_R \cdot x = F_1 a + F_2 b + F_3 \cdot c$

$x = \frac{5600 \cdot 150 + 8900 \cdot 330 + 6300 \cdot 440}{20\,800} = \frac{840\,000 + 2\,937\,000 + 2\,772\,000}{20\,800}$

$= 314{,}8\ mm$

1.56



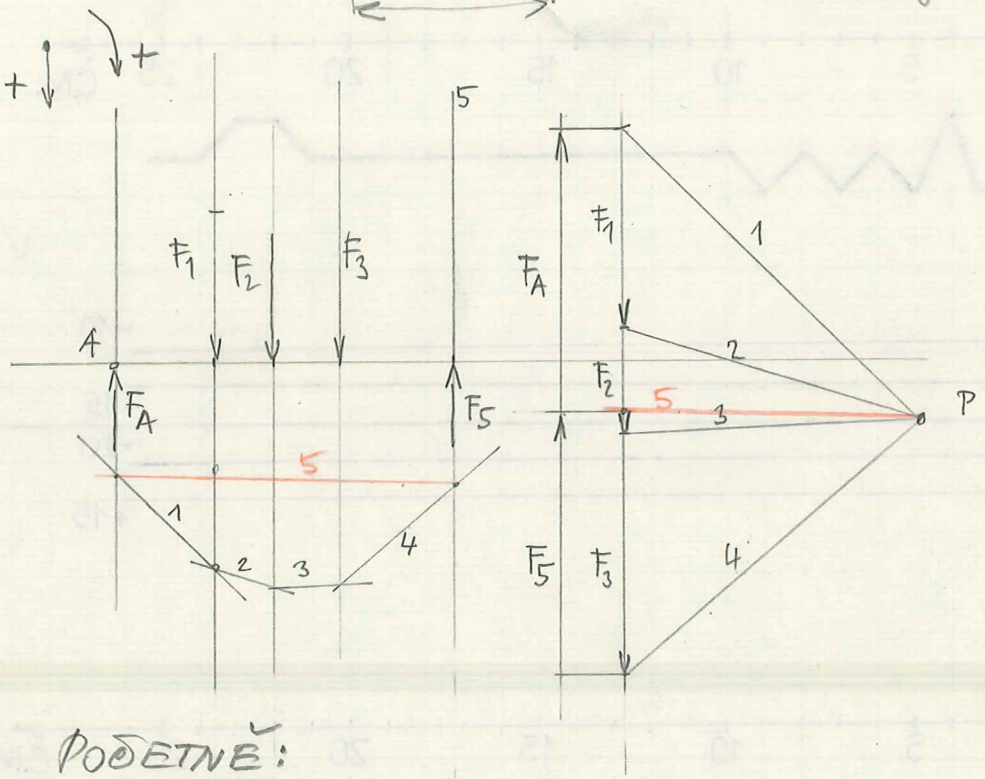
$F_1 = 260\,N$
$F_2 = 140\,N$
$F_3 = 320\,N$
$a = 130\,mm$
$b = 210\,mm$
$c = 150\,mm$
$d = 240\,mm$

F_4 a F_5 je vyvinována pro F_1, F_2, F_3

~~F_4~~ působí v A

F_5 leží na přímce 5

Grafická a početní



$F_A = 370\,N$
$F_5 = 340\,N$

POČETNĚ:

x: --- 0

y: $\Sigma F_{iy} = 0$ --- $-F_A + F_1 + F_2 + F_3 - F_5 = 0$

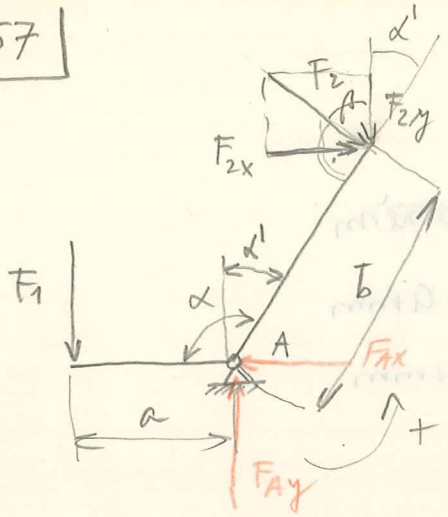
M_A: $\Sigma M_i = 0$ --- $F_1 \cdot a + F_2 \cdot b + F_3 (b + d - c) - F_5 (b + d) = 0$

$$F_5 = \frac{F_1 \cdot a + F_2 b + F_3 (b + d - c)}{b + d} = \frac{260 \cdot 130 + 140 \cdot 210 + 320 (210 + 240 - 1}{210 + 240}$$

$$= \frac{33\,800 + 29\,400 + 96\,000}{450} = 353{,}7\,N$$

$$F_A = F_1 + F_2 + F_3 - F_5 = 260 + 140 + 320 - 353{,}7 = 366{,}3$$

1.57



$F_1 = 500\ N$ $\quad a = 200\ mm$ $\quad F_{Ax} = ?$

$F_2 = ?$ $\quad b = 500\ mm$ $\quad F_{Bx} = ?$

$\alpha = 120°$

$\Sigma M_i = 0 \ \dots \ F_1 \cdot a - F_2 \cdot b = 0$

$$F_2 = F_1 \frac{a}{b} = 500 \frac{200}{500} = \underline{\underline{200\ N}}$$

$\alpha' = \alpha - 90 = 120 - 90 = 30°$

$\beta = 90 - \alpha' = 60°$

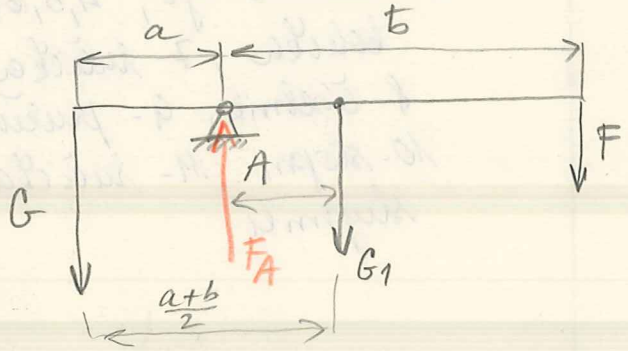
$F_{2y} = F_2 \cos\beta = 200 \cdot \cos 60° = \underline{\underline{100\ N}}$

$F_{2x} = F_2 \sin\beta = 200 \cdot \sin 60° = \underline{\underline{173{,}2\ N}}$

X: $\quad F_{Ax} = F_{2x} = \underline{\underline{-173{,}2\ N}}$

$F_{Ay} = F_1 + F_{2y} = 500 + 100 = \underline{\underline{600\ N}}$

1.58



$G = 1620\ N$

$G_1 = 90\ N$ – vlastní tíha ojce

$F = ?$ $\quad$ Rovnováha

$a = 300\ mm$

$b = 1700\ mm$

$F_A = ?$

$M_A \ \dots \ \Sigma M_i = 0$

$$G \cdot a - G_1 \left(\frac{a+b}{2} - a \right) - F \cdot b = 0$$

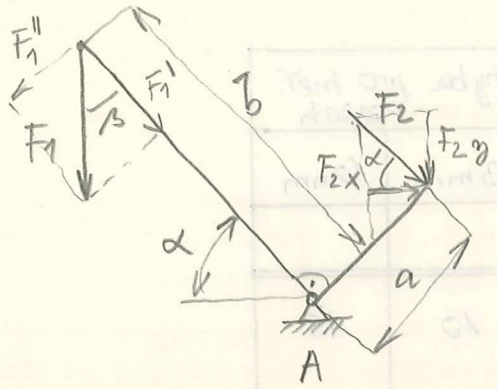
$$F = \frac{G \cdot a - G_1 \left(\frac{a+b}{2} - a \right)}{b} =$$

65000

$$= \frac{1620 \cdot 300 - 90 \left(\frac{2000}{2} - 300 \right)}{1700} = \underline{\underline{248{,}8\ N}}$$

$$F_A = G + G_1 + F = 1620 + 90 + 248{,}8 = \underline{\underline{1958{,}8\ N}}$$

1.59



$F_1 = ?$ Rovnováha

$F_2 = 1500\,N$

$a = 250\,mm$

$b = 800\,mm$

$\alpha = 30°$

$\beta = 60°$

$F_A = ?$

$\sum M_i = 0, \curvearrowleft \quad F_1'' \cdot b - F_2 a = 0$

$$F_1'' = F_2 \frac{a}{b} = 1500 \frac{250}{800} = \underline{468{,}75\,N}$$

$$F_1 = \frac{F_1''}{\sin\beta} = \frac{468{,}75}{\sin 60°} = \underline{541{,}3\,N}$$

$F_{2x} = F_2 \cos\alpha = 1500 \cdot \cos 30° = 1299\,N$

$F_{2y} = F_2 \sin\alpha = 1500 \cdot \sin 30° = 750\,N$

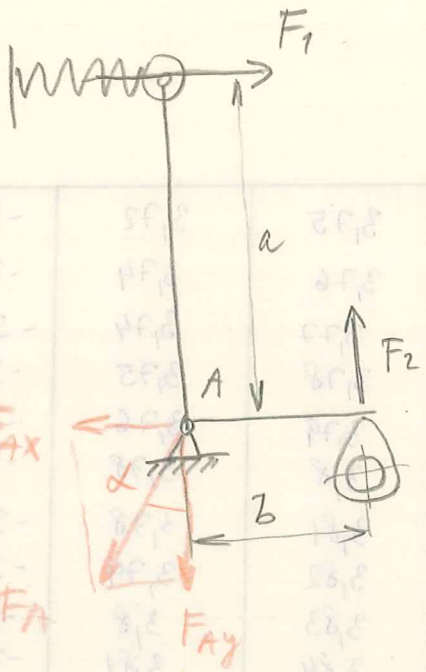
$F_{Ax} = F_{2x} = \underline{1299\,N}$

$F_{Ay} = F_1 + F_{2y} = 541{,}3 + 750 = \underline{1291{,}3\,N}$

$F_A = \sqrt{F_{Ax}^2 + F_{Ay}^2} = \sqrt{1299^2 + 1291{,}3^2} = \underline{1831{,}6\,N}$

1830

1.60



$F_1 = 320\,N$; $a = 120\,mm$

$F_2 = ?$; $b = 85\,mm$ Ravnoteža

$F_A = ?$

$\alpha = ?$ s pravcem a

$\Sigma M_i = 0$ --- $F_1 \cdot a - F_2 \cdot b = 0$

$F_2 = F_1 \frac{a}{b} = 320 \frac{120}{85} = \underline{451,7\,N}$

x: $\Sigma F_{ix} = 0$ --- $F_1 - F_{Ax} = 0$

y: $\Sigma F_{iy} = 0$ --- $F_2 - F_{Ay} = 0$

$F_{Ax} = F_1 = 320\,N$

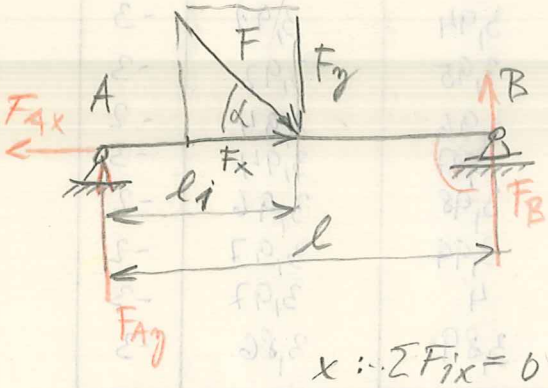
$F_{Ay} = F_2 = 451,7\,N$

$F_A = \sqrt{F_{Ax}^2 + F_{Ay}^2} = \sqrt{320^2 + 451,7^2} = \underline{553,6\,N}$

$tg\,\alpha = \frac{F_{Ax}}{F_{Ay}} = \frac{320}{451,7} = 0,708$

$\underline{\alpha = 35,31°}$

1.61 Nosnik A, B $F = 410\,N$ $l = 710\,mm$ $\alpha = 38°$ $l_1 = 360\,mm$



$F_A = ?$; $F_B = ?$

$F_x = F\cos\alpha = 410\cos 38° = 323\,N$

$F_y = F\sin\alpha = 410\sin 38° = 252,4\,N$

x: $\Sigma F_{ix} = 0$ --- $F_x - F_{Ax} = 0$

$F_{Ax} = F_x = \underline{323\,N}$

y: $\Sigma F_{iy} = 0$ --- $F_{Ay} - F_y + F_B = 0$

Π_A: $\Sigma M_i = 0$ --- $F_B \cdot l - F_y \cdot l_1 = 0$

$F_B = F_y \frac{l_1}{l} = 252,4 \frac{360}{710} = \underline{128\,N}$

$F_{Ay} = F_y - F_B = 252,4 - 128 = \underline{124,4\,N}$

$F_A = \sqrt{F_{Ax}^2 + F_{Ay}^2} = \sqrt{323^2 + 124,4^2} = \underline{346,1\,N}$

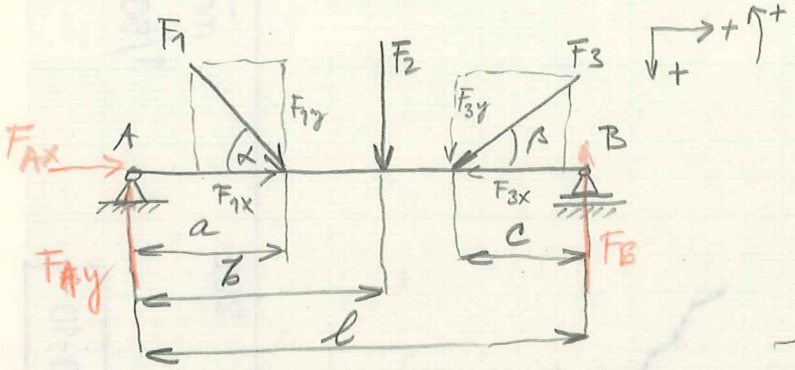
1.63

$F_A = ?$ $F_B = ?$

$F_1 = 830\,N$ $F_2 = 560\,N$ $F_3 = 980\,N$

$a = 160\,mm$ $b = 310\,mm$ $c = 230\,mm$

$l = 620\,mm$ $\alpha = 62°$ $\beta = 29°$



$F_{1x} = F_1 \cos\alpha = 830 \cos 62° = 389{,}6\,N$

$F_{1y} = F_1 \sin\alpha = 830 \sin 62° = 732{,}8\,N$

$F_{3x} = F_3 \cos\alpha = 980 \cos 29° = 857{,}1\,N$

$F_{3y} = F_3 \sin\alpha = 980 \sin 29° = 475{,}1\,N$

$x: \dots \Sigma F_{ix} = 0 \dots F_{Ax} + F_{1x} - F_{3x} = 0$

$y: \dots \Sigma F_{iy} = 0 \dots -F_{Ay} + F_{1y} + F_2 + F_{3y} - F_B = 0$

$M_A: \dots \Sigma M_{iA} = 0 \dots -F_{1y} \cdot a - F_2 \cdot b - F_{3y}(l-c) + F_B l = 0$

$F_{Ax} = F_{3x} - F_{1x} = 857{,}1 - 389{,}6 = \underline{467{,}5\,N}$

$F_B = \dfrac{F_{1y} \cdot a + F_2 b + F_{3y}(l-c)}{l} =$

$= \dfrac{732{,}8 \cdot 160 + 560 \cdot 310 + 475{,}1 \cdot 390}{620} =$

$= \dfrac{117\,248 + 173\,600 + 185\,289}{620} = \underline{767{,}9\,N}$

$F_{Ay} = F_{1y} + F_2 + F_{3y} - F_B = 732{,}8 + 560 + 475{,}1 - 767{,}9 =$

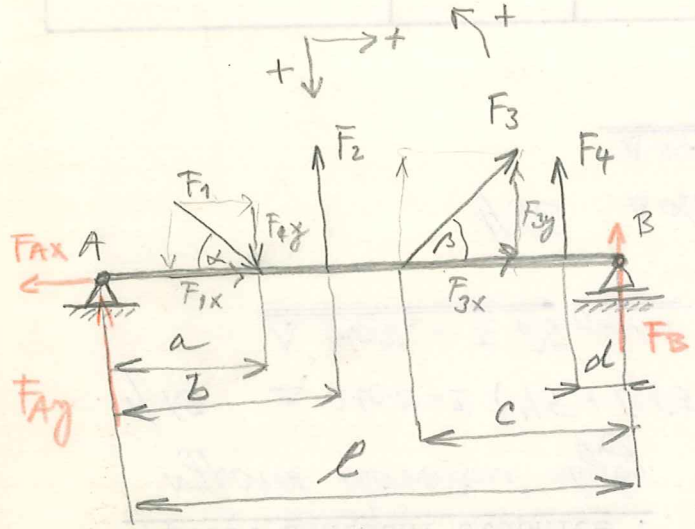
$= \underline{1000\,N}$

$F_A = \sqrt{F_{Ax}^2 + F_{Ay}^2} = \sqrt{467{,}5^2 + 1000^2} = \underline{1104\,N}$

1.64 $F_A = ?$, $F_B = ?$

$F_1 = 1580\,N$, $F_2 = 940\,N$, $F_3 = 1216\,N$

$F_4 = 1420\,N$, $\alpha = 64°$, $\beta = 44°$



$a = 1{,}6\,m$

$b = 2{,}9\,m$

$c = 3{,}1\,m$

$d = 1{,}7\,m$

$\ell = 7{,}6\,m$

$F_{1x} = F_1 \cos\alpha = 1580 \cos 64° = 692{,}6\,N$

$F_{1y} = F_1 \sin\alpha = 1580 \sin 64° = 1420\,N$

$F_{3x} = F_3 \cos\beta = 1216 \cos 44° = 874{,}7\,N$

$F_{3y} = F_3 \sin\beta = 1216 \sin 44° = 844{,}7\,N$

$X: \Sigma F_{xi} = 0 \dots -F_{AX} + F_{1x} + F_{3x} = 0$

$y: \Sigma F_{yi} = 0 \dots -F_{Ay} + F_{1y} - F_2 - F_{3y} - F_4 - F_B = 0$

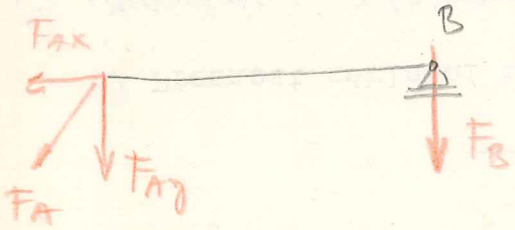
$M_A: \Sigma M_i = 0 \dots -F_{1y} \cdot a + F_2 b + F_{3y}(\ell - c) + F_4(\ell - d) + F_B \ell = 0$

$F_{AX} = F_{1x} + F_{3x} = 692{,}6 + 874{,}7 = 1567{,}3\,N$

$$F_B = \frac{+F_{1y} \cdot a + F_2 b + F_{3y}(\ell - c) + F_4(\ell - d)}{\ell} =$$

$$= \frac{+1420 \cdot 1{,}6 + 940 \cdot 2{,}9 + 844{,}7 \cdot 4{,}5 + 1420 \cdot 5{,}9}{7{,}6} =$$

$$= \frac{2272 - 2726 - 3801{,}1 - 8378}{7{,}6} = -1662{,}2\,N$$

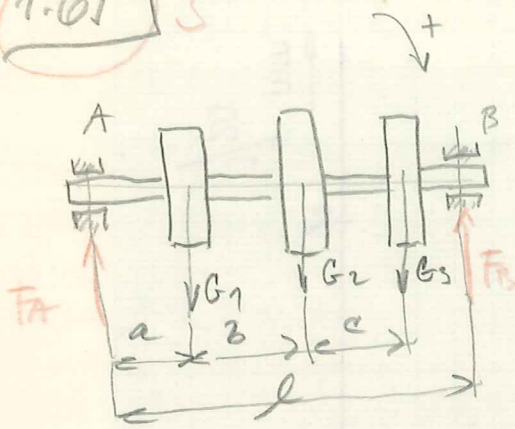


$F_{Ay} = F_{1y} - F_2 - F_{3y} - F_4 + F_B =$

$= 1420 - 940 - 844{,}7 - 1420 + 1662{,}2 = -122{,}5$

$F_A = \sqrt{F_{AX}^2 + F_{Ay}^2} = \sqrt{1567{,}3^2 + 122{,}5^2} = 1572$

1.65 | 3



$G_1 = 1200\,N$; $G_2 = 1850\,N$; $G_3 = 1430\,N$

$l = 7{,}2\,m$

$a = 2{,}1\,m$

$b = 1{,}8\,m$

$c = 2{,}2\,m$

$d_B = d_A = 80\,mm$

$f_c = 0{,}12$

$F_A = ?$

$F_B = ?$

$M_B = ?$

$x: \Sigma F_{ix} = 0 \quad \dots \quad 0$

$y: \Sigma F_{iy} = 0 \quad \dots \quad F_A - G_1 - G_2 - G_3 + F_B = 0$

$M: -\Sigma M_i = 0 \quad \dots \quad G_1 a + G_2(a+b) + G_3(a+b+c) - F_B l = 0$

$$F_B = \frac{G_1 a + G_2(a+b) + G_3(a+b+c)}{l}$$

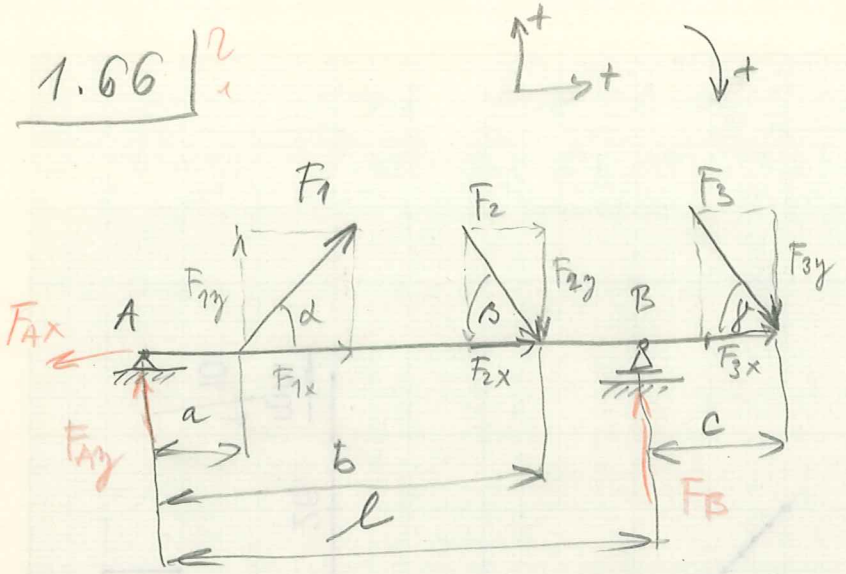
$$= \frac{1200 \cdot 2{,}1 + 1850(2{,}1+1{,}8) + 1430(2{,}1+1{,}8+2{,}2)}{7{,}2}$$

$$= \frac{2520 + 7215 + 8723}{7{,}2} = \underline{2563{,}6\,N}$$

$$F_A = G_1 + G_2 + G_3 - F_B = 1200 + 1850 + 1430 - 2563{,}6$$

$$= \underline{1916{,}4\,N}$$

1.66



$F_1 = 2580\,N$
$F_2 = 3780\,N$
$F_3 = 1640\,N$
$\alpha = 37°$
$\beta = 52°$
$\mu = 65°$
$a = 1{,}2\,m$
$b = 3{,}4\,m$
$c = 1{,}8\,m$
$l = 4{,}5\,m$

$F_A = ?,\ F_B = ?$

$F_{1x} = F_1 \cos\alpha = 2580 \cos 37° = 2020{,}5\,N$
$F_{1y} = F_1 \sin\alpha = 2580 \sin 37° = 1522{,}5\,N$
$F_{2x} = F_2 \cos\beta = 3780 \cos 52° = 2327{,}2\,N$
$F_{2y} = F_2 \sin\beta = 3780 \sin 52° = 2978{,}6\,N$
$F_{3x} = F_3 \cos\alpha = 1640 \cdot \cos 65° = 693\,N$
$F_{3y} = F_3 \sin\alpha = 1640 \sin 65° = 1486{,}3\,N$

$X: \Sigma F_{ix} = 0 \dots -F_{Ax} + F_{1x} + F_{2x} + F_{3x} = 0$

$y: \Sigma F_{iy} = 0 \dots F_{Ay} + F_{1y} - F_{2y} + F_B - F_{3y} = 0$

$M: \Sigma M_i = 0 \dots -F_{1y}\,a + F_{2y}\,b - F_B\,l + F_{3y}(l+c) = 0$

$$F_B = \frac{-F_{1y}\,a + F_{2y}\,b + F_{3y}(l+c)}{l} =$$

$$= \frac{-1522{,}5 \cdot 1{,}2 + 2978{,}6 \cdot 3{,}4 + 1486{,}3 \cdot 6{,}3}{4{,}5} =$$

$$= \frac{-1827 + 10127{,}2 + 9363{,}7}{4{,}5} = 3925{,}3\,N$$

F_{Ay}

$F_{Ay} = -F_{1y} + F_{2y} - F_B + F_{3y} = -1522{,}5 + 2978{,}6 - 3656{,}6 + 1486{,}3 =$

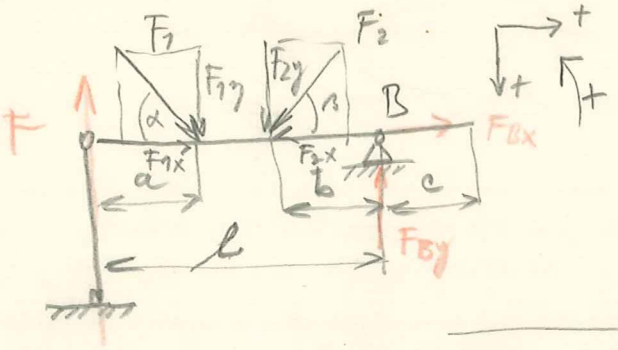
$= -714{,}2\,N$

$F_{Ax} = F_{1x} + F_{2x} + F_{3x} = 2020{,}5 + 2327{,}2 + 693 = 5040{,}7\,N$

$F_A = \sqrt{F_{Ax}^2 + F_{Ay}^2} = \sqrt{5040{,}7^2 + 714{,}2^2} = 5091\,N$

1.67

$F_B = ?$, $F = ?$ $F_1 = 220\,N$, $F_2 = 390\,N$



$\alpha = 55°$, $\beta = 34°$

$a = 720\,mm$

$b = 520\,mm$

$c = 390\,mm$

$l = 1640\,mm$

$F_{1x} = F_1 \cos\alpha = 220 \cos 55° = 126{,}2\,N$

$F_{1y} = F_1 \sin\alpha = 220 \cdot \sin 55° = 180{,}2\,N$

$F_{2x} = F_2 \cos\beta = 390 \cos 34° = 323{,}3\,N$

$F_{2y} = F_2 \sin\beta = 390 \sin 34° = 218{,}1\,N$

$x: \Sigma F_{ix} = 0 \dots F_{1x} - F_{2x} + F_{Bx} = 0$

$y: \Sigma F_{iy} = 0 \dots -F + F_{1y} + F_{2y} - F_{By} = 0$

$M: \Sigma M_{iB} = 0 \dots F_{2y} \cdot b + F_{1y} \cdot (l-a) + F \cdot l = 0$

$$F = \frac{F_{2y} b + F_{1y}(l-a)}{l} = \frac{218{,}1 \cdot 520 + 180{,}2 \cdot 920}{1640}$$

$$= \frac{113\,412 + 165\,784}{1640} = \underline{\underline{170{,}2\,N}}$$

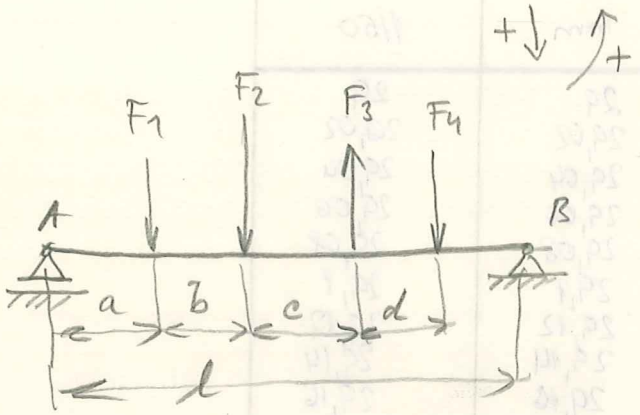
$F_{Bx} = F_{2x} - F_{1x} = 323{,}3 - 126{,}2 = \underline{\underline{197{,}1\,N}}$

$F_{By} = -F + F_{1y} + F_{2y} = -170{,}2 + 180{,}2 + 218{,}1 = \underline{\underline{228{,}1\,N}}$

$F_B = \sqrt{F_{Bx}^2 + F_{By}^2} = \sqrt{197{,}1^2 + 228{,}1^2} = \underline{\underline{301{,}5\,N}}$

1.68

$F_1 = 1350\,N$, $F_2 = 1620\,N$, $F_3 = 2140\,N$
$F_4 = 1280\,N$
$a = 1{,}4\,m$
$b = 1{,}8\,m$
$c = 2{,}1\,m$
$d = 0{,}8\,m$
$l = 6{,}9\,m$



$x \dots \Sigma F_{ix} = 0 \quad \dots\dots \quad 0$

$y \dots \Sigma F_{iy} = 0 \quad \dots \quad -F_A + F_1 + F_2 - F_3 + F_4 - F_B = 0$

$M \dots \Sigma M_{iA} = 0 \quad \dots\dots \quad -F_1 a - F_2(a+b) + F_3(a+b+c) - F_4(a+b+c+d) + F_B \cdot l = 0$

$$F_B = \frac{F_1 a + F_2(a+b) - F_3(a+b+c) + F_4(a+b+c+d)}{l} =$$

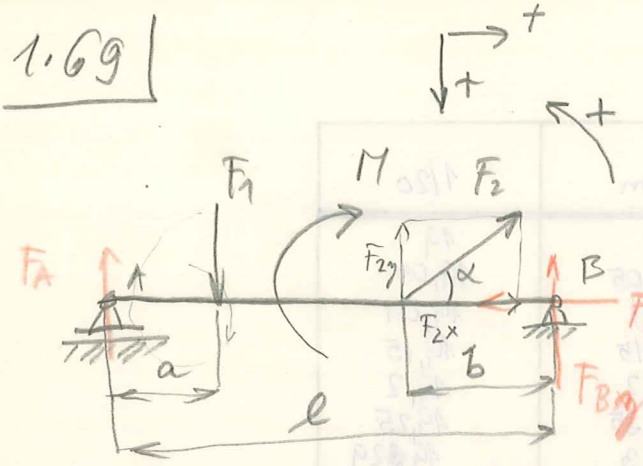
$$= \frac{1350 \cdot 1{,}4 + 1620 \cdot 3{,}2 - 2140 \cdot 5{,}13 + 1280 \cdot 6{,}1}{6{,}9} =$$

$$= \frac{1890 + 5184 - 11342 + 7808}{6{,}9} = \underline{\underline{513\,N}}$$

$$F_A = F_1 + F_2 - F_3 + F_4 - F_B = 1350 + 1620 - 2140 + 1280 - 513 =$$

$$= \underline{\underline{1597\,N}}$$

1.69



$F_1 = 540\,N$ $\alpha = 58°$

$F_2 = 380\,N$ $M = 890\,Nm$

$a = 2{,}36\,m$

$b = 3{,}15\,m$

$\ell = 6{,}72\,m$

$F_{2x} = F_2 \cos\alpha = 380 \cdot \cos 58° = 201{,}3$

$F_{2y} = F_2 \sin\alpha = 380 \cdot \sin 58° = 322{,}2$

$x: \Sigma F_{ix} = 0 \quad \dots \quad F_{2x} - F_{Bx} = 0$

$y: \Sigma F_{iy} = 0 \quad \dots \quad -F_A + F_1 - F_{2y} - F_{By} = 0$

$M_A \cdot \Sigma M_i = 0 \quad \dots \quad -F_1 \cdot a - M + F_{2y}(\ell - b) + F_{By}\,\ell = 0$

$$F_{By} = \frac{F_1 a + M - F_{2y}(\ell - b)}{\ell} = \frac{540 \cdot 2{,}36 + 890 - 322{,}2 \cdot 3{,}57}{6{,}72} =$$

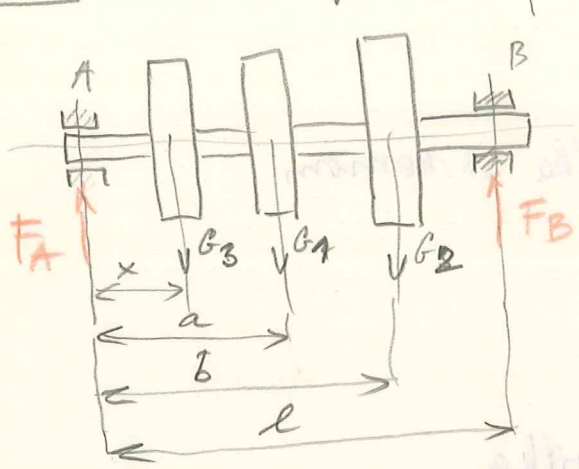
$$= \frac{1274{,}4 + 890 - 1150{,}2}{6{,}72} = \underline{150{,}9\,N}$$

$$F_{Bx} = F_{2x} = \underline{201{,}3\,N}$$

$$F_B = \sqrt{F_{Bx}^2 + F_{By}^2} = \sqrt{201{,}3^2 + 150{,}9^2} = \underline{251{,}5\,N}$$

$$F_A = F_1 - F_{2y} - F_{By} = 540 - 322{,}2 - 150{,}9 = \underline{66{,}9\,N}$$

1.70



$G_1 = 510\,N$ $\quad a = 260\,mm$

$G_2 = 340\,N$ $\quad b = 570\,mm$

$G_3 = 420\,N$ $\quad l = 630\,mm$

$x = ?$ $\quad F_A = F_B$

$x: \Sigma F_{ix} = 0 \quad \dots \quad 0$

$y: \Sigma F_{iy} = 0 \quad \dots \quad -F_A + G_3 + G_1 + G_2 - F_B = 0$

$M: \Sigma M_A = 0 \quad \dots \quad -G_3 \cdot x - G_1 \cdot a - G_2 \cdot b + F_B \cdot l = 0$

$$2F_A = G_3 + G_1 + G_2$$

$$F_A = F_B = \frac{G_3 + G_1 + G_2}{2} = \frac{420 + 510 + 340}{2} = 635\,N$$

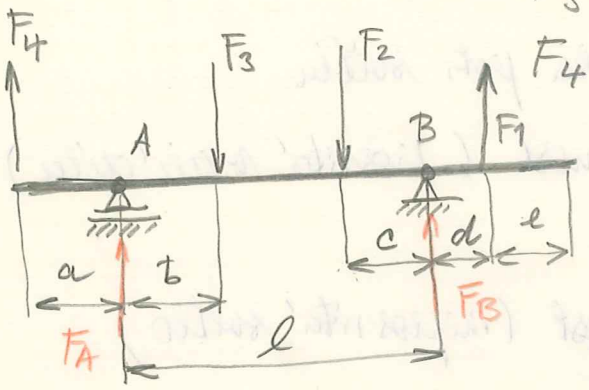
$$x = \frac{-G_1 a - G_2 b + F_B l}{G_3} = \frac{-510 \cdot 260 - 340 \cdot 570 + 635 \cdot 630}{420}$$

$$= \frac{-132\,600 - 193\,800 + 400\,050}{420} = 175{,}4\,mm$$

1.71

$+\uparrow$ $\circlearrowright+$

$F_1 = 100\,N$ $\quad a = 200\,mm$
$F_2 = 300\,N$ $\quad b = 100\,mm$
$F_3 = 200\,N$ $\quad c = 200\,mm$
$F_4 = 100\,N$ $\quad \ell = 500\,mm$
$\quad d = 200\,mm$
$\quad e = 180\,mm$



$F_A = ?$
$F_B = ?$

$X: -\Sigma F_{ix} = 0$ ---- 0

$y: -\Sigma F_{iy} = 0$ --- $F_4 + F_A - F_3 - F_2 + F_B + F_1 = 0$

$M: -\Sigma M_A = 0$ --- $-F_4 \cdot a - F_3 \cdot b - F_2(\ell - c) + F_B \cdot \ell + F_1(\ell + d) = 0$

$$F_B = \frac{F_4 a + F_3 b + F_2(\ell - c) - F_1(\ell + d)}{\ell} =$$

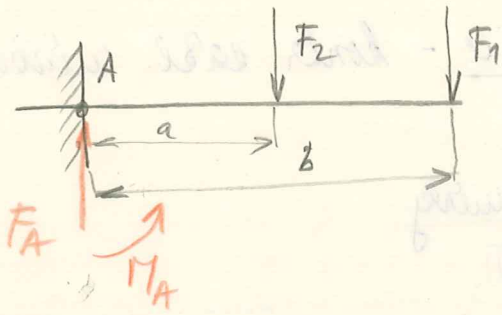
$$= \frac{100 \cdot 200 + 200 \cdot 100 + 300(500 - 200) - 100(500 + 200)}{500} =$$

$$= \frac{20000 + 20000 + 90000 - 70000}{500} = \underline{\underline{120\,N}}$$

$$F_A = -F_4 + F_3 + F_2 - F_B - F_1 = -100 + 200 + 300 - 120 - 100$$

$$= \underline{\underline{180\,N}}$$

1.72



$F_1 = 540\,N$
$F_2 = 380\,N$
$a = 410\,mm$
$b = 670\,mm$

$X: \Sigma F_{ix} = 0 \dots\dots 0$

$y: \Sigma F_{iy} = 0 \dots\dots F_A - F_2 - F_1 = 0$

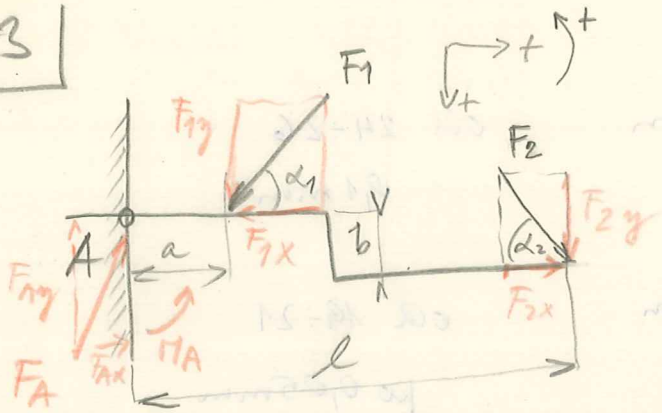
$F_A = F_2 + F_1 = 380 + 540 = \underline{\underline{920\,N}}$

$M: \Sigma M_A = 0 \dots M_A - F_2 \cdot a - F_1 b = 0$

$M_A = F_2 \cdot a + F_1 \cdot b = 380 \cdot 410 + 540 \cdot 670 = 517600\,Nmm$

$= \underline{\underline{517{,}6\,Nm}}$

1.73



$F_1 = 960\,N$
$F_2 = 670\,N$
$\alpha_1 = 62°$
$\alpha_2 = 31°$
$a = 2{,}1\,m$
$b = 1{,}2\,m$
$l = 3{,}8\,m$

$F_{1x} = F_1 \cos\alpha_1 = 960 \cos 62° = 450{,}6\,N$
$F_{1y} = F_1 \sin\alpha_1 = 960 \sin 62° = 847{,}6\,N$
$F_{2x} = F_2 \cos\alpha_2 = 670 \cos 31° = 574{,}3\,N$
$F_{2y} = F_2 \sin\alpha_2 = 670 \sin 31° = 345\,N$

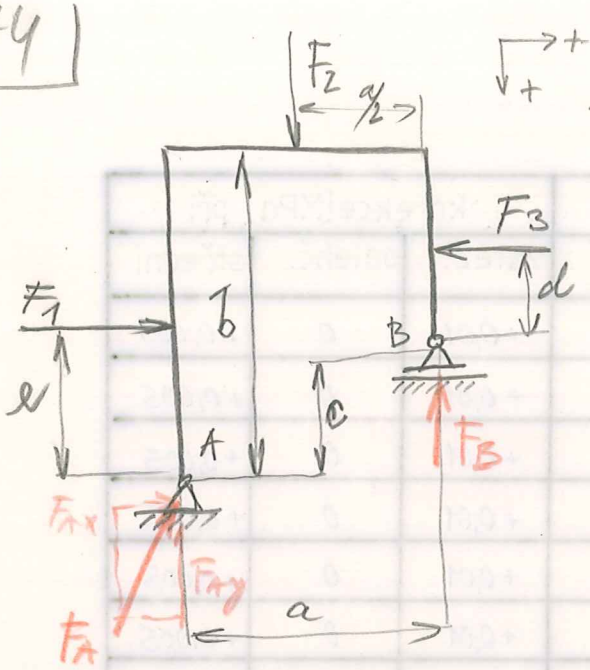
$X: \Sigma F_{ix} = 0 \quad --- \quad F_{Ax} - F_{1x} + F_{2x} = 0$
$y: \Sigma F_{iy} = 0 \quad --- \quad -F_{Ay} + F_{1y} + F_{2y} = 0$
$M: \Sigma M_i = 0 \quad --- \quad M_A - F_{1y} \cdot a - F_{2y} \cdot l + F_{2x} b = 0$

$M_A = F_{1y} a + F_{2y} \cdot l - F_{2x} b =$
$= 847{,}6 \cdot 2{,}1 + 345 \cdot 3{,}8 - 574{,}3 \cdot 1{,}2 =$
$= 1779{,}9 + 1311 - 689{,}2 = \underline{2401{,}7\,Nm}$

$F_{Ax} = F_{2x} - F_{1x} = 574{,}3 - 450{,}6 = \underline{123{,}7\,N}$
$F_{Ay} = F_{1y} + F_{2y} = 847{,}6 + 345 = \underline{1192{,}6}$

$F_A = \sqrt{F_{Ax}^2 + F_{Ay}^2} = \sqrt{123{,}7^2 + 1192{,}6^2} = \underline{1198{,}9\,N}$

1.74



$F_1 = 860\,N$

$F_2 = 670\,N$

$F_3 = 1140\,N$

$a = 1{,}4\,m$

$b = 2{,}7\,m$

$c = 0{,}6\,m$

$d = 1{,}2\,m$

$e = 1{,}5\,m$

$x: \Sigma F_{ix} \dots + F_{Ax} + F_1 - F_3 = 0$

$y: \Sigma F_{iy} \dots - F_{Ay} + F_2 - F_B = 0$

$M_A: \Sigma M_{iA} \dots - F_1 \cdot e - F_2 \frac{a}{2} + F_3 (c+d) + F_B \cdot a = 0$

$$F_B = \frac{F_1 e + F_2 \frac{a}{2} - F_3 (c+d)}{a}$$

$$= \frac{860 \cdot 1{,}5 + 670 \cdot 0{,}7 - 1140 \cdot 1{,}8}{1{,}4} =$$

$$= \frac{1290 + 469 - 2052}{1{,}4} = -209{,}2\,N \downarrow F_B$$

$F_{Ay} = F_2 - F_B = 670 - (-209{,}2) = 879{,}2\,N$

$F_{Ax} = -F_1 + F_3 = -860 + 1140 = 280\,N$

$F_A = \sqrt{F_{Ax}^2 + F_{Ay}^2} = \sqrt{280^2 + 879{,}2^2} = 922{,}7\,N$